Pre-Positioning and Local-Purchasing for Emergency Operations Under Budget and Supply Uncertainty

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Joint with Jeannette Song (Duke) and Scott Webster (ASU)

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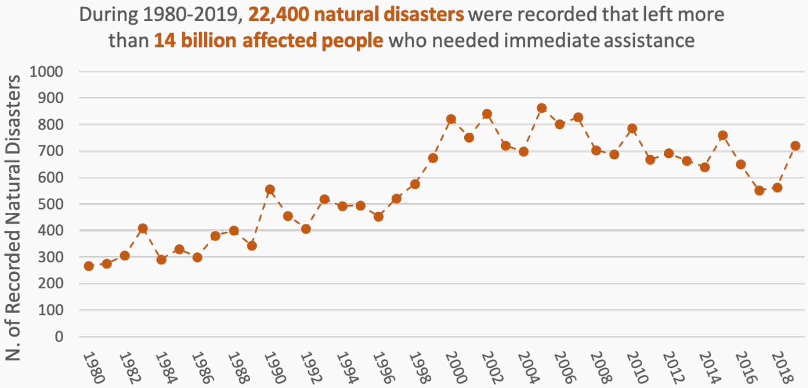
MIT Center for Transportation & Logistics

Background


During 1980-2019, 22,400 natural disasters were recorded that left more than 14 billion affected people who needed immediate assistance
N. of Recorded Natural Disasters
1000
900
800
700
600
500
400
300
200
100
0
1980
1982
1984
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
2008
2010
2012
2014
2016
2018

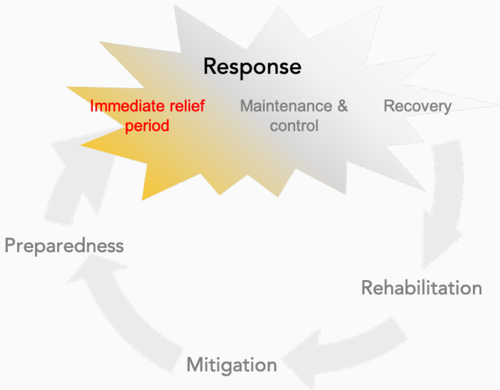
Background

The frequency, the intensity and the severity of disasters are in a fast-growing trend.

Background

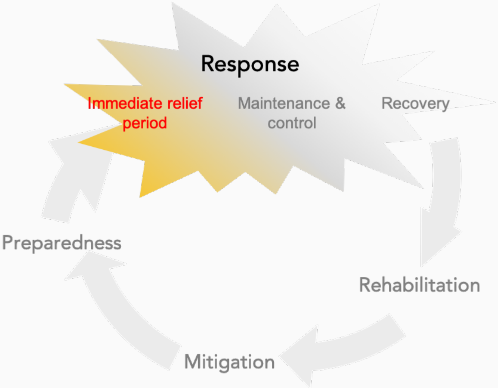
Effective response to rapid-onset disasters (e.g., earthquake) is extremely challenging due to the **unknowns**; when? where? how many?

Background



Immediate relief period is the most critical stage of response operations.

Background



Primary goals: (i) quick response, and (ii) securing enough supply of life-saving items (e.g., water, sanitation, and food).

Background: Common Practice

International HOs typically adopt two policies to fulfill the demand: proactive or reactive.

Background: Common Practice

Proactive policy: Prepositioning inventory at strategic locations e.g., World Vision International (4 sites), UN Humanitarian Response Depot (6 sites).

Background: Common Practice

Advantages: Enough time to buy and store the selected relief items, at a low purchase price with assurance of quality.

Background: Common Practice

Challenges: Demand uncertainty, ignoring political, economic, demographic and environmental realities of the affected areas.

Background: Common Practice

Reactive policy: Using local supply aftermath of a disaster

Background: Common Practice

Advantages: More precise demand estimation, culturally accepted products, and stimulation of the local economy

Background: Common Practice

Challenges: Supply uncertainty due to e.g., collapsing the banking system, or competition among HOs.

Question

Optimal level of prepo, if prioritizing reactive supply?

CRS

- US-based international HO operating in 110 countries
- 5,000 employees around the world, revenue $\sim$ \$1 billion
- contributes in development programs
- provides relief in emergencies
- at time of disaster, CRS distributes a subset of 20 commodities e.g., blankets, jerrycans, hygiene kits, household/kitchen kits, and prepackaged food kits.
- CRS stores its inventories in 2 strategically located warehouses; Philippines and Madagascar, as well as UNHRD's Dubai, Panama and Accra sites.
- informed us that almost no HO has a guideline to determine prepo level.

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CATHOLIC RELIEF SERVICES faith. action. results.

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Landing Price

Internal audit of four HOs (CRS, CARE, Mercy Corps, and WVI) shows:

Financial Limitation

Donors are less interested in providing funds for inventory in advance of a disaster.

Existing Moldes

Existing Models

Location-allocation models: determining the location of major (or temporary) DCs and prepositioned inventory levels at various warehouses.

Recent examples: Duran et al. 2011; Chakravarty 2014; Charles et al. 2016; Paul and MacDonald 2016; Jahre et al. 2016; Noham and Tzur 2017; Ni et al. 2018; Tofighi et al. 2016; Erbeyoğlu and Bilge 2020; Simchi-Levi et al. 2019; Ye et al. 2020.

Existing Models

Network design and LMD: proposing models to improve material flow, and so consider reliability of transportation network, vehicle routing, and aspects of equity.

Few examples: Barbarosoglu and Arda (2004); Balcik et al. (2008); Huang et al. (2012); Afshar and Haghani (2012); Noyan et al. (2016); Vanajakumari et al. (2016); Dalal and Üster (2017); Mills et al. (2018).

Existing Models

Two-stage stochastic mixed integer programing;

the first stage decisions are made before the realization of random variables, and the second stage recourse variables are determined after realization of the random variables.

1. ignore local supply (and its uncertainty) as well as budget uncertainty
2. too complex and data-intensive
3. require data that is available after disaster strikes
4. scenario-based stochastic models lack a realistic base
5. all insights are based on numerical illustrations

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Contribution

What is an optimal level of prepo stock of a single- (and multi-)relief items, in recognition of uncertainty in

demand,

local supply,

time to next disaster, and

funds for local spend?

And, what if we have access to limited data?

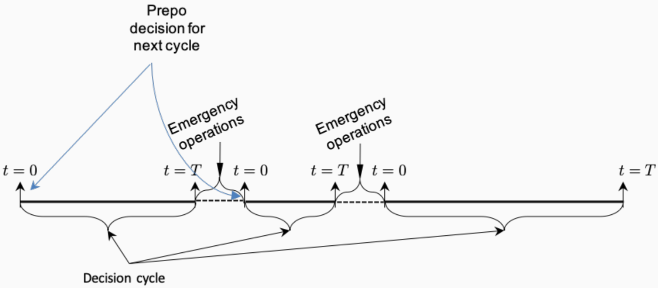
Model

Setting

General assumptions and notation:

- c is the unit purchasing and transportation cost of prepo.
- Prepo item is more expensive ($c \geq \alpha c$), and we normalize $c = 1$.

Setting



- The time between two events (T) is random.
- Demand (D), and supply (Q) are random, independent of time, and uncorrelated.

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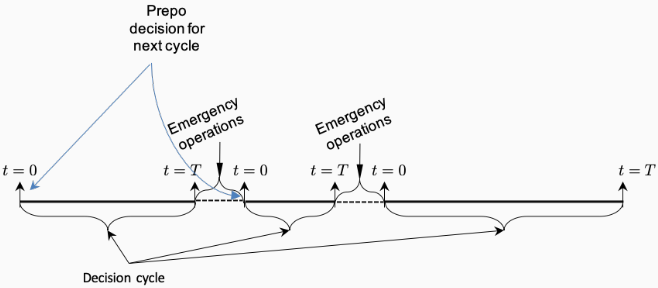

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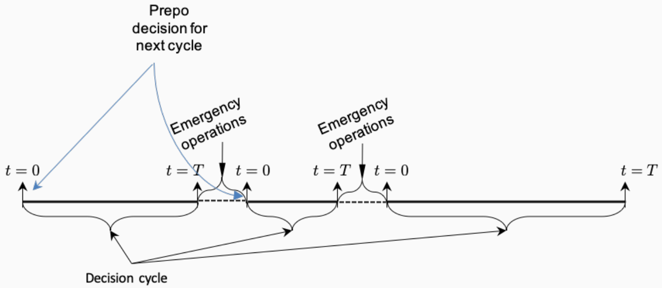

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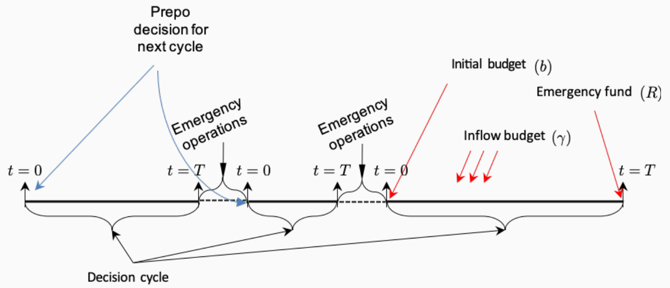

- *b* includes carryover prepo stock from previous cycle converted into money value.
- Inflow of funds (γ) is at a fixed rate per period, during each cycle.
- Random emergency fund is received right after disaster occurs (R).

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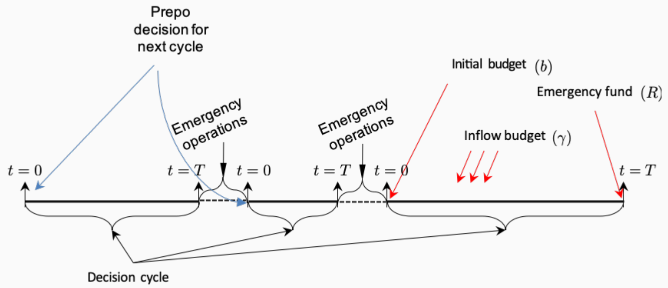

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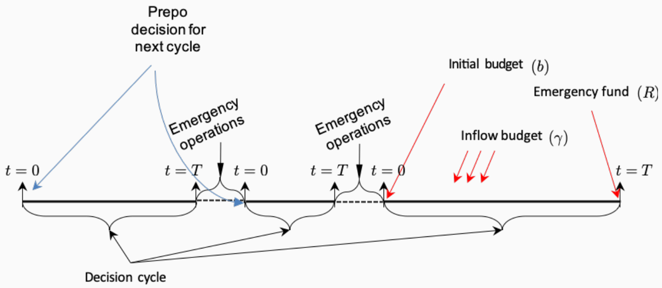
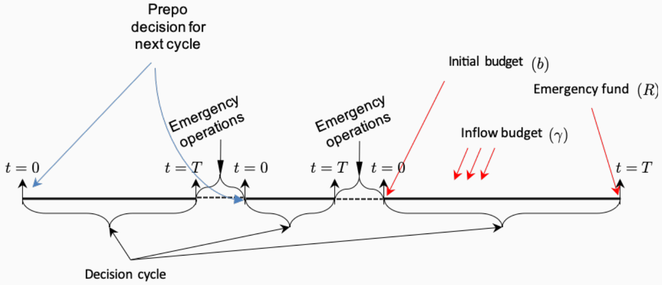


- Available local budget $(\hat{b}(x, R, T))$ includes the sum of initial budget, cash inflow, and emergency fund minus prepo cost.
- The random local purchase quantity is $L(x) = \min\{D, Q, \hat{b}(x, R, T)/\alpha\}$.

Setting



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Setting

Expected cost during a cycle is

$$C(x) = E\left[\underbrace{\alpha \min\left\{D, Q, \frac{\hat{b}(x,R,T)}{\alpha}\right\}}_{\text{re-active purchases}} + \overbrace{\min\{x, S(x)\}}^{\text{pro-active purchases used during IRP}} + \overbrace{ixT}^{\text{holding cost}} + \underbrace{v\left(S(x) - x\right)^{+}}_{\text{global shortage cost}}\right]$$

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Uncertain local shortage is

$$S(x) = \left(D - \min\left\{Q, \frac{\hat{b}(x,R,T)}{\alpha}\right\}\right)^+$$

Setting

The problem to solve is

$$\min_{x \geq 0} C(x) = \alpha\mu_D + ix\mu_T + (1-\alpha)E[S(x)] + (v-1)E[(S(x)-x)^+]$$

$$x \leq b.$$

Setting

Three events:

Ω_1 both demand and supply are more than available local budget.

Ω_2 demand is larger than the sum of local budget and prepo, and supply is more than local budget (i.e., insufficient budget and prepo).

Ω_3 demand is more than the sum of local supply and prepo, and supply is less than local budget (i.e., insufficient local supply).

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Optimal Prepo Stock

$C(x)$ is convex, and optimal prepo stock is

$$x^* = \min\left\{(x^\circ)^+, b\right\}$$

where x° is the optimal unconstrained prepo stock obtained from a unique solution to first-order condition.

The core tradeoff relies on marginal cost vs. marginal saving, which depend on marginal local shortage ($\frac{1}{\alpha}P[\Omega_1(x)]$) and marginal global shortage ($\frac{1-\alpha}{\alpha}P[\Omega_2(x)] - P[\Omega_3(x)]$) .

Tradeoff

- marginal local shortage: the rate at which local shortage increases per unit increase in prepo (under event Ω_1).
- marginal global shortage: the rate at which global shortage increases per unit increase in prepo (under event Ω_2), and per unit decrease in prepo (under event Ω_3).

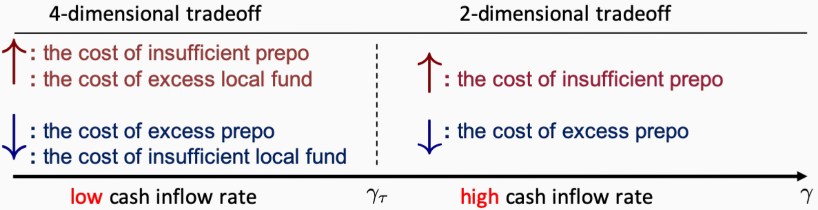
Comparative Statics

Directional impact of [variable, if increasing]	Sufficient budget	Insufficient budget
Disaster frequency	$\nearrow$	$\nearrow$ $\searrow$
Shortage cost	$\nearrow$	$\nearrow$
Holding cost	$\searrow$	$\searrow$
Average local supply	$\searrow$	$\searrow$
Uncertainty of emergency funds	Unaffected	$\nearrow$ $\searrow$
Average emergency funds	Unaffected	$\nearrow$ $\searrow$
Volatility of disaster frequency	Unaffected	$\nearrow$ $\searrow$
Cash inflow	Unaffected	$\nearrow$
Cost of local supply	Unaffected	$\nearrow$ $\searrow$
Initial budget	$\nearrow$	$\nearrow$
Cost per unit of prepo	$\searrow$	$\searrow$
Demand or supply uncertainty	$\nearrow$ If critical, $\searrow$ otherwise	$\nearrow$ $\searrow$
Average demand	$\nearrow$	$\nearrow$ $\searrow$
Effective **approximate solution**	We found simple approximate solution.	

A relief item is labeled critical if the optimal unconstrained prepo stock (x_+) exceeds the expected mismatch between demand and local supply (i.e., $x_+ > \mu_D - \mu_Q$).

Source of Complexity

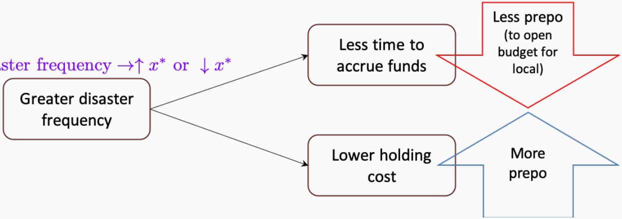
Example: For a given b, there is a threshold cash inflow (γ_τ) above which FOC shows a Newsvendor-type tradeoff:



Source of Complexity

Why can't we have a determined direction when budget is limited?

Disaster frequency $\rightarrow \uparrow x^*$ or $\downarrow x^*$



Source of Complexity

When budget is limited, HOs need more information or a clear strategy to determine optimal prepo level.

Auxiliary Model

Consider an auxiliary model where *local budget* constraint is relaxed i.e., $\min\left\{Q, \frac{\hat{b}(x,R,T)}{\alpha}\right\}$ is replaced with Q, in $S(x)$ (i.e., only limited by the local supply.) Then, the cost function is

$$C_a(x) = \alpha\mu_D + i\mu_T x + (1-\alpha)\,E\left[(D-Q)^+\right] + (v-1)\,E\left[(D-Q-x)^+\right],$$

which conforms to the classic Newsvendor structure.

Approximate Solution: UB

If sufficient local budget is available:

$$\bar{x}^* = \arg\min_{x \leq b} \left\{ C_a(x) = \min\{(x_+)^+, b\} \right\}$$

where,

- $\star$ x_+ balances the cost of inventory against the relative shortage.
- $\star$ The same insights gained from NV problem apply with random D is replaced by the net demand $(D - Q)$.
- $\star$ UB is exact when the budget is above a threshold.

Approximate Solution: LB

The lower bound is

$$\underline{x}^* = \min\{x_+, b\}$$

where,

$$x_+ = \max\{\bar{m}_c(x) - \underline{m}_s(x) \geq 0, x \geq 0\}$$

$\bar{m}_c(x)$: the upper-bound of marginal cost when increasing prepo,

$\underline{m}_s(x)$: the lower bound of the marginal savings when increasing prepo

Numerical Experiments

Based on CRS & Synthetic Data

- Facility in Lipa city
- Serving Laos, Indonesia, Guinea, Philippines, Myanmar, Bangladesh, Vietnam, and Cambodia
- June 2006 – June 2018
- Date, type, magnitude (no. of people affected), and location of disaster
- 66 rapid-onset disasters that affected >1,000 people
- Hygiene kit \$35 $\rightarrow$ \$50 with transportation, required for every 5 people

- Stochastic optimization with 100,000 trials per simulation via Analytic Solver Platform
- Over 6,000 numerical experiments

General Patterns

Basic patterns in prepo and expected cost curves are relatively stable across changes in shortage cost rate, and local supply cost ratio.

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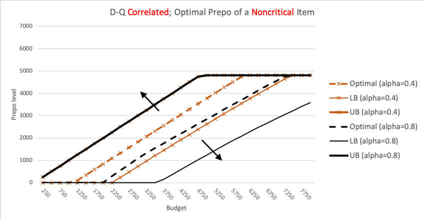
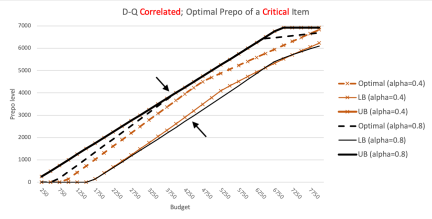
Basic patterns in prepo and expected cost curves are relatively stable across changes in shortage cost rate, and local supply cost ratio.

We find higher marginal value of budget for the items with higher shortage cost, and diminishing return to increases in budget.

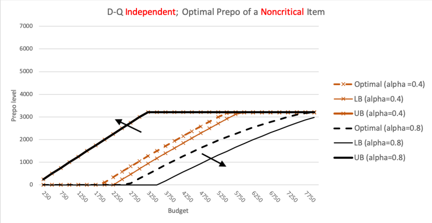
Exact vs. Approximate

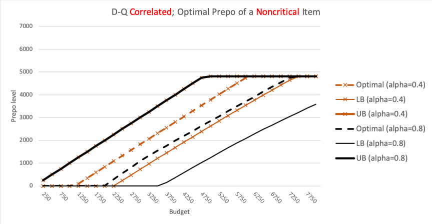




Optimal prepo and LB/UB are larger, and the difference between LB and UB is less if item is more critical.

Exact vs. Approximate





Optimal prepo and LB/UB are larger, and the difference between LB and UB is less if D and Q are negatively correlated.

Exact vs. Approximate

Optimal prepo tends to be much closer to LB for independent Q–D, and somewhat closer to UB for negatively correlated Q–D.

Exact vs. Approximate

<table>
<tr><td></td><td colspan="2">High local price</td><td colspan="2">Low local price</td></tr>
<tr><td></td><td>High shortage cost</td><td>Low shortage cost</td><td>High shortage cost</td><td>Low shortage cost</td></tr>
<tr><td>Low emergency fund</td><td>Close to UB</td><td>Close to UB if D-Q correlated</td><td colspan="2">Close to LB if D-Q independent, but close to UB if correlated</td></tr>
<tr><td>High emergency fund</td><td colspan="4">Close to UB</td></tr>
</table>

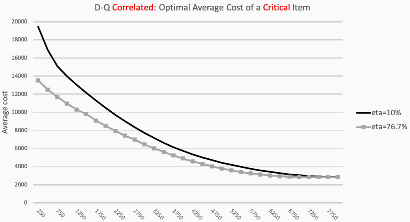
Emergency Funds

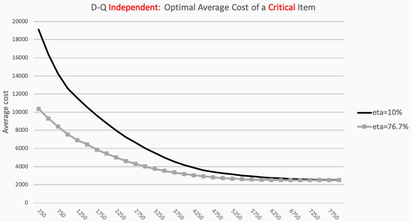
When is emergency fund more useful?

- In general: as emergency funds increase $\Rightarrow$ prepo $\Uparrow$ and expected cost $\Downarrow$.
- Additional dollar in the emergency fund is valuable as it aligns with demand; It generates more value for independent D–Q.

Emergency Funds

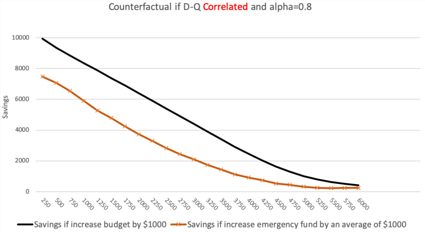
The value of emergency funds decreases when budget increases.

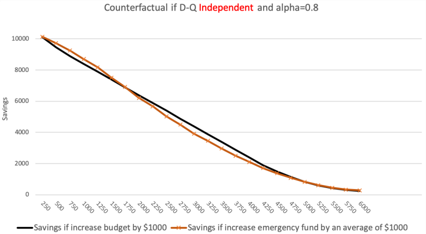




Additional dollar in the initial budget provides more flexibility, and in practice, relying on emergency fund is too risky.

Emergency Funds





Whether an increase in emergency fund adds more or less value depends on parameters value; neither funding source dominates the other.

Emergency Funds

Quick remark

$\rightarrow$ If reactive, emergency fund might be efficient in some conditions.

$\rightarrow$ If proactive, emergency fund is almost always less efficient than pre-disaster investment.

Contribution to Practice

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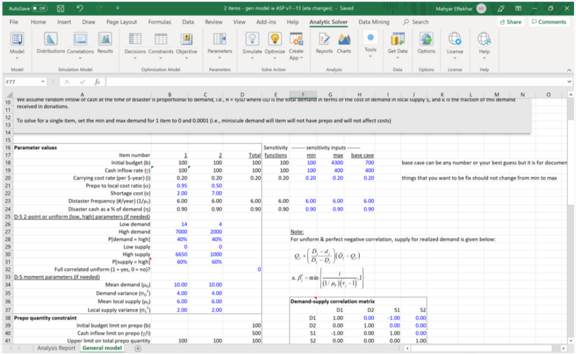
Results have been shared with a number of large HOs.

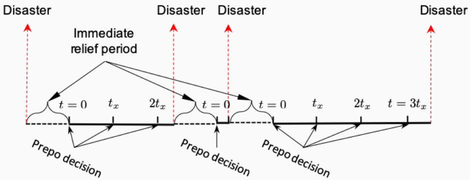


We also created a simple Excel-based calculator, and are developing a widget.

Extensions

Dynamic Policy

The prepo level is reviewed on a periodic basis:



- ⋆ If inflow cash is too small or too large, the static and dynamic cases are equivalent; set prepo at the beginning and leave it fixed.
- ⋆ Otherwise, for a given budget, dynamic prepo level is dominated by static prepo level, and is nondecreasing in review period length.
- ⋆ All of the directional effects identified continue to hold for the dynamic model.

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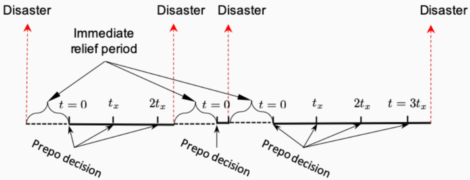

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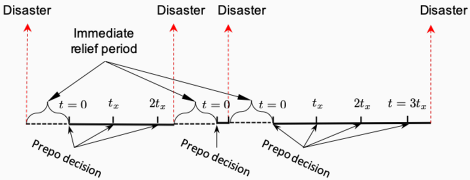

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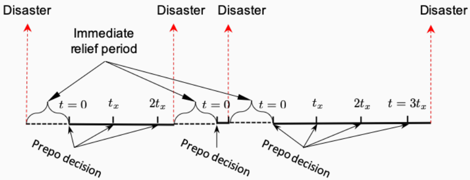

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Prepo Landing Price is Lower (Proactive Model)

USAID

Ebola outbreak in Liberia

"Everyone needed the same things (disinfectant, thermometers, etc.), leading to a significant supply shortage and price surge in the field. It was cheaper to buy it internationally compared to the local market."

Prepo Landing Price is Lower (Proactive Model)

Objective cost function will be different but there are some similarities in results:

- the cost function is convex in prepo
- similar budget threshold that delineates a structural change in the prepo optimization problem

Prepo Landing Price is Lower (Proactive Model)

Directional impact of [variable, if increasing]	Local supply is cheaper		Prepo is cheaper	
	Sufficient budget	Insufficient budget	Sufficient budget	Insufficient budget
Disaster frequency	↗	↗ ↘	↗	↗ ↘
Shortage cost	↗	↗	↗	↗
Holding cost	↘	↘	↘	↘
Average local supply	↘	↘	↘	↘
Uncertainty of emergency funds	Unaffected	↗ ↘	Unaffected	↗ ↘
Average emergency funds	Unaffected	↗ ↘	Unaffected	↗ ↘
Volatility of disaster frequency	Unaffected	↗ ↘	Unaffected	↗ ↘
Cash inflow	Unaffected	↗	Unaffected	↗ ↘
Cost of local supply	Unaffected	↗ ↘	↗	↗ ↘
Initial budget	↗	↗	↗	↗ ↘
Cost per unit of prepo	↘	↘	↘	↗ ↘
Demand or supply uncertainty	↗	↗ ↘	↗	↗ ↘
Effective **approximate solution**	We found simple approximate solution.		We have not been able to find it.	

Prepo Landing Price is Lower (Proactive Model)

Differences:

- the marginal value of prepo is nonmonotonic when $\alpha > 1$
    1. the simpler LB/UB cannot be found
    2. some unexpected comparative statics are concluded

Prepo Landing Price is Lower (Proactive Model)

Example: $b \uparrow \Rightarrow x^* \uparrow$ or $x^* \downarrow$

Why prepo decreases? The value of additional prepo is linked to the value of replacing a unit of more expensive reactive stock and the value of reducing very costly unsatisfied demand. When budget increases, the likelihood of unsatisfied demand decreases, so prepo decreases. This is for cases when you have large local supply.

Multi-item

- Multiple item when some are cheaper if prepo
- Price volatility for all items
- Supply, demand, budget uncertainty
- We drive structural properties of optimal solutions

Thank you for your attention!