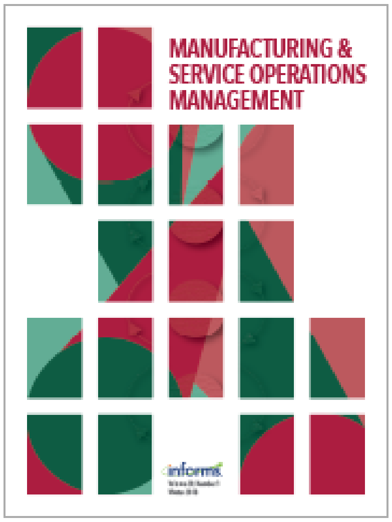

Manufacturing & Service Operations Management

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informs
http://pubsonline.informs.org/journal/msom
MANUFACTURING & SERVICE OPERATIONS MANAGEMENT
Articles in Advance, pp. 1–18
ISSN 1523-4614 (print), ISSN 1526-5498 (online)

Prepositioning and Local Purchasing for Emergency Operations Under Budget, Demand, and Supply Uncertainty

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Received: June 5, 2019
Revised: February 4, 2020; July 21, 2020; September 2, 2020
Accepted: September 11, 2020
Published Online in Articles in Advance: March 18, 2021

https://doi.org/10.1287/msom.2020.0956

Abstract. ***Problem definition***: Considering a mix of prepositioning and local purchasing, common to cover humanitarian demands in the aftermath of a rapid-onset disaster, we propose policies to determine preposition stock. These formulations are developed in the presence of demand, budget, and local supply uncertainties and for single-items delivery. ***Academic/practical relevance***: The immediate period aftermath of a disaster is the most crucial period during which humanitarian organizations must supply relief items to beneficiaries. Yet, because of many unknowns such as time, place, and magnitude of a disaster, supply management is a significant challenge, and these decisions are made intuitively. The features and complexities we examine have not been studied in the literature. ***Methodology***: We derive properties of the optimal solution, identify exact solution methods, and determine approximate methods that are easy to implement. ***Results***: We (i) characterize the interplay of supply, demand, and budget uncertainties, as well as the impact of product characteristics on optimal prepo stock levels; (ii) show in what conditions the prepo stock is a simple newsvendor solution; and (iii) discuss the value of emergency funds. ***Managerial implications***: We show that budget level is a key determinant of the optimal policy. When it is above a threshold, inventory increases in disaster frequency and severity, but the reverse is true otherwise. When budget is limited, the rate of savings from improved forecasts is amplified (attenuated) for critical (noncritical) items, reflecting opposing directional effects of mismatch cost and cost of insufficient funding. Our model can also be used to estimate the value of initiatives to mitigate constraints on local spend (e.g., a line of credit underwritten by large donors that is available during the immediate relief period).

Supplemental Material: The online appendix is available at https://doi.org/10.1287/msom.2020.0956.

Keywords: emergency relief operations • supply management policies • preposition stock • reactive stock

1. Introduction

This paper concentrates on the management of supplies for the *immediate relief period* (IRP) of rapid-onset natural disasters (e.g., earthquake, tsunami), which every year affect around 160 million people worldwide (World Health Organization 2020). The IRP is the first and most crucial stage of an emergency response, where life-saving efforts are the primary focus, and is succeeded by a *maintenance and control* stage, where the situation stabilizes, and a *recovery* stage. During this short period, the local infrastructure may be damaged, and the abilities of local actors might be severely restricted. The humanitarian organizations' (HOs') main objective is quick response and providing adequate supply of life-saving items (e.g., water, sanitation, hygiene—also known as WASH—food, shelter, and medicine). Yet, assurance of adequate supply is highly challenging because of the unpredictability of demand, financial limitations (Balcik and Beamon 2008), and local supply uncertainty (De la Torre et al. 2012). The purpose of this paper is to develop an analytical framework that accounts for the multiple uncertainties to aid humanitarians' decision making. Based on our intense interactions with managers at a few large international HOs, we synthesize the common practice and then, develop a model to capture the special features that managers face in practice. Considering uncertainty in demand, local supply, and budget, we characterize optimal inventory policy for a wide range of scenarios. Furthermore, we compare our results with those coming from a simple newsvendor model to present a deeper theoretical discussion.

1.1. Practice

There are two common supply management approaches for the IRP. The first is *proactive*, under which HOs purchase relief items from international suppliers, preposition this inventory (*prepo* stock) in strategically located distribution centers (Duran et al. 2011), and ship the items to the affected areas only after disaster strikes. World Vision International, for example, prepositions relief items in four locations, from which the items are packaged and transported anywhere in the world within 72 hours. The Federal Emergency Management Agency has nine distribution centers to support the continental United States and its territories. The World Food Program manages the United Nations Humanitarian Response Depots in seven locations where large HOs such as Catholic Relief Services (CRS), the Cooperative for Assistance and Relief Everywhere (CARE), and Oxfam store items.

The advantage of prepo stock is that HOs have enough time to buy and store the selected relief items at a low purchase price with assurance of quality. However, because of the unpredictability of disasters, it is nearly impossible to preposition the right items at the right quantity. Also, in some situations, inventory holding cost (including obsolescence and opportunity cost of capital) can be high (Acimovic and Goentzel 2016). Furthermore, to avoid delays, HOs often have to transport items by air (e.g., if ground transportation is more than a two-day transit time to the affected area), which is very expensive. An internal audit of four large international HOs (i.e., CRS, CARE, Mercy Corps, and World Vision International) revealed that ordering and purchasing, shipment from suppliers to HO's warehouses, and warehousing costs consist of 50% of total supply chain costs under a proactive approach (40%, 2%, and 8%, respectively), whereas transportation cost from the HO's warehouse to an affected area makes up 46% of the overall supply chain expenditures.[1] Because of high transport costs, the total *landed cost* (i.e., purchase, warehousing, and transport) is high. Thus, although a proactive approach has been proven successful for some organizations, it is overall an extremely expensive approach (Kunz et al. 2014). Furthermore, donors tend to be less interested in providing funds for inventory in advance of a disaster, requiring that an HO procure stock from its limited reserve accounts with the hope of replenishing from donor contributions in response to an emergency.

The second approach is *reactive*, under which HOs purchase inventory (called *reactive stock*) after a disaster strikes from suppliers in or near the affected area and distribute among beneficiaries using relatively inexpensive transport modes such as truck. There are several advantages of reactive stock. First, demand assessment at the time of the purchase decision can be done much more accurately. Second, although purchasing from local suppliers may be subject to competitive bidding because of the presence of many HOs in the affected area, hence leading to high purchase prices, the total landed cost of the locally acquired items is usually less than the prepo stock because of lower transport costs. A manager we interviewed at CRS states: "The unit cost of most items like kitchen sets, clean up supplies, hygiene supplies, etc. is cheaper while purchasing locally or regionally." He provides examples of the tsunami response where a lot of items were procured from the region and similarly for the Pakistan earthquake. The cost advantage is reinforced in our interview of a manager at the US Agency for International Development (USAID): "In general, purchasing items from the field makes a lot more sense because it is much cheaper" (Tang 2006, p. 38). Third, reactive stock offers local economic and cultural benefits, as local purchasing helps to dampen the negative economic shock from the disaster and helps to speed economic recovery of the area, and it has the advantage of providing culturally acceptable products (i.e., products that are familiar to the local population (Duran et al. 2011)). A manager at CRS shared an example where plastic latrine caps from prepo were shipped into Aceh for the tsunamic response, and CRS only discovered later that the local population would not use latrines. A similar example was shared by a USAID expert where clothes made in Israel were shipped into Iraq and were not accepted by the local population. On the other hand, a disaster in large magnitude may lead to collapse of the local banking system or destroy the local supply base. "We experienced supply shortage in Liberia during Ebola, as a result of the main import companies having an issue to bring the items in country." Also, even if local supplies are available for purchase, the quantity may be insufficient, and quality may be substandard. In spite of these drawbacks, HOs generally prioritize the use of reactive stock over prepo. Executives of those HOs that we interviewed stressed that they prefer to keep a minimum level of prepo, essentially as a backup. Yet, there is no guideline to determine the optimal level of prepo inventory, and experienced managers make these decisions only based on their intuition.

We discuss related literature in the next section and present the model in Section 3. In Section 4, we propose and analyze an optimal static policy (i.e., prepo remains fixed until the next disaster occurs), and in Section 5, we propose and analyze an optimal dynamic policy (i.e., prepo is updated on a periodic basis). Section 6 presents numerical illustrations. Although our main focus is on policies where the local market offers a lower price, we briefly discuss the similarities and differences between this case and the case where local price is greater than the cost of a prepo item (Section 7). Section 8 concludes. Proofs and derivations are available in the online supplement.

2. Positioning in the Literature

Supply management in emergency relief operations has drawn significant attention in the Operations Management and Management Science literature, most of which focuses on the proactive approach. Topics include optimal prepositioning locations and inventory level (de Treville et al. 2006, Balcik and Beamon 2008, Duran et al. 2011, Manoj et al. 2016, Dalal and Üster 2017), response capacity (Salmeron and Apte 2010, Kunz et al. 2014, Toyasaki et al. 2017, Ni et al. 2018), and location-allocation models (Mete and Zabinsky 2010). These studies typically analyze a two-stage linear stochastic program, with the first stage focusing on facility location and the second stage considering inventory level (Balcik and Beamon 2008, Rawls and Turnquist 2010, Galindo and Batta 2013, Klibi et al. 2013, Tofighi et al. 2016). With the exception of Rawls and Turnquist (2010), most studies examine a case where demand uncertainty and transportation network unreliability may exist. Because of the complexity of these settings, managerial insights are often obtained through numerical experiments. Recognizing the limitations of the numerical approach, Campbell and Jones (2011) develop a newsvendor-type model for identifying a supply point and level of inventory. Supply is uncertain because a supply point may be damaged during a disaster, and the probability of damage is estimated using the distance between the site and the disaster. A more recent example is Simchi-Levi et al. (2019), which studies the problem of prepositioning inventory in a setting where inventory of medical countermeasures is prepositioned to protect infected population against bioattacks. They propose a two-stage robust optimization model that minimizes inventory and life loss costs.

Our paper is distinct from this stream of literature in three main respects. First, in addition to supply and demand uncertainty, we introduce budget uncertainty (i.e., the amount of funds available for local purchases during the IRP) specific to humanitarian settings that has been largely ignored. Natarajan and Swaminathan (2014) consider the impact of budget uncertainty on procurement efficiency where demand is predictable and unmet demand is completely backlogged. In their model, the budget sizes are known, but the time of receipt is uncertain. Second, motivated by practice, we consider the presence of a local source with uncertain supply. Except for Duran et al. (2011) that considers the possibility of local supply, a common assumption is that HOs' preference is to supply relief items only from prepo stock. However, the HOs' executives who we interviewed emphasized the opposite direction: they prefer to fulfill the emergency demand during IRP from the local markets first and then cover the additional demands from prepo stock. We develop analytical guidelines on how to effectively use the limited financial resources to balance the use of prepo and reactive stocks. Finally, we extend our model and develop the setting of a dynamic policy (i.e., the management is allowed to update prepo quantity at fixed time intervals before the occurrence of a disaster).

It is worth underlying that demand uncertainty and supply disruption have long been studied in the Operations Management and Management Science literature (see Snyder et al. 2016 for a review). Similar to the proactive approach in humanitarian settings, a common strategy of commercial firms to hedge against supply disruption and random demand surges is to maintain stockpiles (Sheffi 2005, Liu et al. 2016) whose size and source of supply are determined based on a cost-benefit analysis. For instance, Tomlin (2006) shows that a risk-neutral firm will pursue a pure disruption management strategy by carrying inventory and single sourcing from the reliable supplier. To cope with supply uncertainty, Tomlin and Snyder (2007) propose keeping an evolving level of safety stock (i.e., increasing the level of safety stock when supply uncertainty grows), and Tang (2006, p. 38) suggests "storing some inventories" at strategic locations to be shared by multiple partners. Liu et al. (2016) explain that stockpiles are not beneficial until a supply disruption or demand surge occurs. Considering a fast-moving commodity, they suggest a policy that allows virtual stockpile pooling to minimize the overall inventory holding costs. In this stream of literature, the most relevant work to ours is Huang et al. (2016), who study joint reactive capacity and safety stock policies in anticipation of sudden demand surges. Considering several aspects of demand surges (duration, intensity, compactness, volatility, and frequency), they propose a policy to minimize the long-run average expenditures under a fixed service level. Nevertheless, our model significantly differs from theirs. (1) In Huang et al. (2016), safety stock is prioritized, and reactive capacity serves as the second source that is the opposite of our setting. (2) In our model, reactive stock is purchased from local suppliers, and its landing costs less than prepo. (3) We consider local supply uncertainty, budget uncertainty, and budget limitation.

3. Model

We analyze the case of a single relief item such as a kit of essential items (food or medicine) for survival during the IRP. Suppose that a relief event has just concluded. The HO sets its target prepo stock, denoted x, at a single distribution center in preparation to cover the demand during the IRP for the next disaster that might hit a particular region of the world. The location of the distribution center is exogenous. A prepo cost cycle starts at the end of a disaster's IRP and ends when the IRP of the next disaster is over (see Figure 1). It consists of two parts, T and Y, where T is the random time from the beginning of the cycle until the next disaster strikes and Y is IRP. During Y, the demand for the item is a random variable D, and the local supply of the item is a random quantity Q. (We note that there may be a positive probability of no local supply; e.g., because a severe event destroys local supply or the product is historically unavailable in the affected region.) Because Y is relatively short and all random variables are realized after this period begins, it is reasonable to assume that all the events during Y happen at one point at the end of T. That is, we can refer to T as the cycle length, and Y degenerates to one point.

The unit purchasing and transport cost of prepo stock is c. Recall from Section 1 that this cost is expensed against the budget when it is used in the field (i.e., c is a one-time expense). In addition, the HO incurs a prepo holding cost at rate i per dollar-period, which is an ongoing expense. To simplify notation, we normalize to $c = 1$. The unit cost of local purchasing and transportation is α multiple of that of prepo stock. Prepo stock is used to protect against the possibility of insufficient local supply within the IRP. Furthermore, prepo stock via air is only used during the IRP purely because it can be moved quickly (at a high cost) to the area of need. In this paper, we assume that the landed cost of prepo c is usually more expensive than the landed cost of locally purchased supplies (i.e., $\alpha < 1$). The unsatisfied demand during the IRP incurs a unit shortage cost v.

At the beginning of the cycle, there is an initial budget b, which also includes the carryover prepo stock from the last cycle. The budget is the initial amount of funds available for investment in prepo and for local spend during the IRP. However, there is an inflow of funds at rate γ per period, and at the time that a disaster hits, a random amount of emergency fund R is received. We allow R to be correlated with D and Q (e.g., emergency funds are higher for disasters with larger magnitude (Eftekhar et al. 2017)). We assume that R, D, and Q are independent of T. Thus, the investment in prepo at the beginning of the cycle is limited by the initial budget (i.e., $x \leq b$). The budget available for local spend is the remaining budget after investment in prepo plus funds received over the cycle: that is, the random upper limit on local spend for a disaster at time T is

$$\hat{b}(x, R, T) = b + \gamma T + R - x. \tag{1}$$

Accordingly, the constraint on the number of units purchased locally, denoted x_L, is $x_L \leq \hat{b}(x, R, T)/\alpha$. Because the HO prioritizes local supply over prepo, the random local purchase quantity is $L(x) = \min\{D, Q, \hat{b}(x, R, T)/\alpha\}$, the random local shortage is $S(x) = (D - \min\{Q, \hat{b}(x, R, T)/\alpha\})^+$, and the random global shortage is $(S(x) - x)^+$. Table 1 summarizes the notation in our model. (Recall that the novel features of our model include the budget b, the choice of local sourcing, and the financial inflow γT and R. If we do not consider the budget limitation and ignore the possibility of local supply (i.e., $b = \infty$ and $Q = 0$), then $L(x) = 0$, $S(x) = D$, and the global shortage is $(D - x)^+$, which is the underage in the classic newsvendor model.)

In summary, there are four distinct cost elements that go into the random cost of a cycle. First, there is the ongoing cost of holding prepo inventory during the cycle, ixT. Second, there is the cost of purchasing and distributing local supply, $\alpha L(x)$. Third, there is the purchase and transport cost of prepo that is used during the IRP, $\min\{x, S(x)\}$. Fourth, there is the cost of unsatisfied demand during the IRP, $v(S(x) - x)^+$. The prepo decision x is constrained by the initial budget, b ($x \leq b$), and the local purchase quantity decision is constrained by the initial budget reduced by prepo investment plus cash inflows during the

Figure 1. (Color online) Prepo Cost Cycle: Decision Maker Decides the Prepo Level at Time $t = 0$ and Disaster Strikes at $t = T$

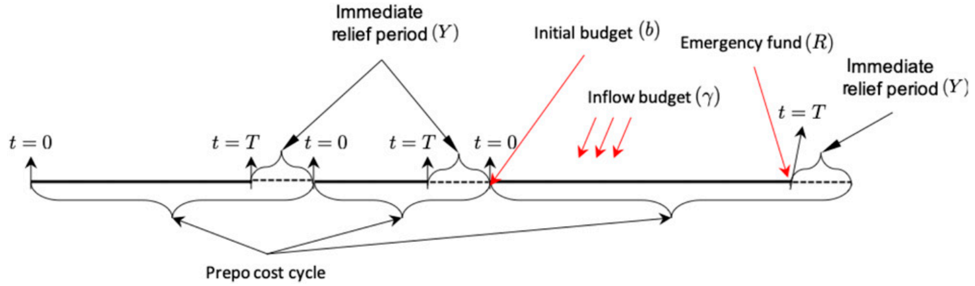


Table 1. Notation Table

Variable/parameter	Description
x	Order-up-to quantity of prepo stock at start of the cycle; the decision variable
c	Cost per unit (including transport) for prepo stock, normalized to 1
α	Ratio of local cost to prepo cost per unit (including transport) or *local supply cost multiple*, $\alpha < 1$
i	Cost per dollar-period for holding prepo stock (e.g., opportunity cost, storage charges, obsolescence)
v	Cost per unit of unsatisfied demand during the immediate relief period, $v > c$
b	Available budget (including current prepo stock) at the start of the cycle
γ	Inflow per period during a cycle (i.e., used for prepo inventory holding cost and purchase of local stock during the immediate relief period)
D	Uncertain demand during immediate relief period
Q	Uncertain local supply during immediate relief period
R	Random cash received at the time of the disaster (emergency fund)
T	Uncertain time to next disaster, which is independent of D, and Q
L	Total quantity purchased locally during immediate relief period
S	Shortage of supply during immediate relief period (because of the lack of supply or budget)
f_j, F_j	Marginal pdf and cdf of $j \in \{D, Q, T\}$
f_{DQR}, F_{DQR}	Joint pdf and cdf of (D, Q, R)
$[\underline{j}, \bar{j}]$	Support of $j \in \{D, Q, R, T\}$, nonnegative, possibly unbounded
μ_j, σ_j^2	Mean and standard deviation of $j \in \{T, D, Q, R, S\}$

Note. cdf, cumulative distribution function; pdf, probability density function.

cycle, $x_L \le \hat{b}(x, R, T)/\alpha$. Thus, the expected cost during a cycle is

$$C(x) = E\Big[iTx + \alpha \min\Big\{D, Q, \frac{b + \gamma T + R - x}{\alpha}\Big\} + \min\{x, S(x)\} + v(S(x) - x)^+\Big]$$
$$= \alpha\mu_D + i\mu_T x + (1-\alpha)E[S(x)] + (v-1)E[(S(x) - x)^+],$$

and the problem at the beginning of the cost cycle is

$$\Pi : \min_{x \le b} C(x).$$

Obviously, if $b = \infty$ and $Q = 0$, then $C(x) = \mu_D + i\mu_T x + (v-1)E[(D-x)^+]$ is the single-source newsvendor cost, and the optimization problem is the classic newsvendor problem without the budget constraint. Note that in a dynamic problem with nonstationary data, the optimization problem Π determines a myopic prepo level that minimizes the expected cost in one cycle. In a dynamic setting with multiple cycles, the initial budget b is the state of the current cycle. The state of the next cycle would be the remaining budget at the end of the current cycle plus a new budget b_+ for the next cycle. When b_+ is significant and data are stationary, it is reasonable to expect that a myopic policy is optimal (i.e., in practice, HOs do not ignore demand of the current disaster and hold inventory for future disasters). Therefore, our findings apply.

4. Analysis

4.1. Optimal Solution

Let Ω denote the support of (D, Q, R, T). Define three events and the partial expectation of a function of random variables

$$\Omega_1(x) := \Big\{(d, q, r, t) : d > \frac{b + \gamma t + r - x}{\alpha}, q > \frac{b + \gamma t + r - x}{\alpha}, (d, q, r, t) \in \Omega\Big\}$$
$$\Omega_2(x) := \Big\{(d, q, r, t) : d > \frac{b + \gamma t + r - x}{\alpha} + x, q > \frac{b + \gamma t + r - x}{\alpha}, (d, q, r, t) \in \Omega\Big\}$$
$$\Omega_3(x) := \Big\{(d, q, r, t) : d > q + x, q \le \frac{b + \gamma t + r - x}{\alpha}, (d, q, r, t) \in \Omega\Big\}$$
$$E[z(D, Q, R, T); \Omega_i(x)] := \int_{\Omega_i(x)} z(d, q, r, t) f_{DQR} \times (d, q, r) f_T(t)\, dd\, dq\, dr\, dt, i = 1, 2, 3.$$

Note that

$$\frac{dE[S(x)]}{dx} = \frac{1}{\alpha} P[\Omega_1(x)] \tag{2}$$
$$\frac{dE[(S(x) - x)^+]}{dx} = \frac{1-\alpha}{\alpha} P[\Omega_2(x)] - P[\Omega_3(x)]. \tag{3}$$

Proposition 1. *$C(x)$ is convex, and optimal prepo stock is*

$$x^* = \min\{(x^o)^+, b\},$$

where $x^o = m^{-1}(0)$ is the optimal unconstrained prepo stock and $m(x) = m_c(x) - m_s(x)$ with

$$m_c(x) = i\mu_T + \frac{1-\alpha}{\alpha}(P[\Omega_1(x)] + (v-1)P[\Omega_2(x)])$$
$$m_s(x) = (v-1)P[\Omega_3(x)].$$

(Proofs and derivations are located in the online appendix.)

The intuition underlying Proposition 1 is as follows. The marginal local shortage (Equation (2)), which appears in the marginal cost function $m_c(x)$, is the rate at which local stock decreases per unit increase in prepo stock under event $\Omega_1(x)$ (i.e., rate $1/\alpha$). On the other hand, the marginal global shortage (Equation (3)), which appears in the marginal savings function $m_s(x)$, is the rate at which total stock decreases per unit increase in prepo stock under event $\Omega_2(x)$ (i.e., rate $1/\alpha - 1$) less the rate at which total stock increases under event $\Omega_3(x)$ (i.e., rate 1). This intuition helps clarify the drivers of the optimal prepo stock quantity in the event that the prepo constraint is nonbinding (denoted x^o).

The next proposition identifies lower bounds (LBs) and upper bounds (UBs) on optimal prepo. These bounds are relatively easy to compute and help to illuminate the offsetting pressures that influence optimal prepo. The bounds rely on additional notation and the definition of an auxiliary model wherein the local budget constraint is relaxed (i.e., $\min\{Q, \frac{\hat{b}(x,R,T)}{\alpha}\}$ is replaced with Q). Our auxiliary model applies to settings where the HO is assured to have enough funds for local spend during the IRP, a condition that holds for larger HOs. The cost function for this auxiliary model is $C_a(x) = \alpha\mu_D + i\mu_T x + (1-\alpha)E[(D-Q)^+] + (v-1)E[(D-Q-x)^+]$, which shows similarity to the newsvendor structure; the optimal solution is

$$\bar{x}^* = \operatorname*{argmin}_{x \le b} C_a(x) = \min\{(x_+)^+, b\}, \tag{4}$$

where

$$x_+ = \bar{F}^{-1}_{D-Q}(\beta^*) \qquad \text{with} \qquad \beta^* = \frac{i\mu_T}{v-1}.$$

(When there is no local source (i.e., $Q = 0$), this is exactly the classic newsvendor solution.) We see that x_+ is fractile $1-\beta^*$ of the random demand net of random local supply and that the optimal shortage probability (β^*) balances the cost rate of excess prepo ($i\mu_T$) with the cost rate of insufficient prepo ($v-1$). Because this solution carries the same form of the newsvendor solution, the same insights gained from the newsvendor problem equally apply here when the random demand in the former is replaced by the net demand $D-Q$. For example, according to Song (1994), x_+ is larger if $D-Q$ is stochastically larger. Assuming D and Q are independent, this condition includes the situations when the distribution of Q (D) is fixed, whereas D (Q) is stochastically larger (smaller). Similarly, a more variable D or Q implies a more variable $D-Q$, which in turn, implies a larger x_+ if β^* is smaller than a threshold; otherwise, x_+ should be lower.

Observe that maximum funding need for local spend (as of the beginning of the cost cycle) is $\max\{\alpha\min\{d,q\} - \gamma t - r : (d,q,r,t) \in \Omega\}$, which because of independence of triple (D,Q,R) and T, can be written as $\max\{\alpha\min\{d,q\} - r : (d,q,r) \in \Omega\} - \gamma\underline{T}$. So, if the budget at the beginning of the cycle is

$$\bar{b} = \max\{\alpha\min\{d,q\} - r : (d,q,r) \in \Omega\} - \gamma\underline{T} + (x_+)^+ \tag{5}$$

or more, then the local spend constraint is not binding. By comparing budget b with threshold budget $\bar{b}$, we can identify conditions wherein problem Π reduces to the auxiliary model. As we see in Proposition 2, the optimal solution to the auxiliary model provides an upper bound on optimal prepo that can be interpreted as a simple newsvendor-based heuristic. This heuristic is exact when the budget is above the characterized threshold; otherwise, the heuristic (i.e., upper-bound formula) is suboptimal. The proposition requires the following additional notation and assumption:

$$\underline{m}_c(x) = i\mu_T$$
$$\bar{m}_s(x) = (v-1)\bar{F}_{D-Q}(x)$$
$$\underline{m}(x) = \underline{m}_c(x) - \bar{m}_s(x)$$
$$x_+ = \underline{m}^{-1}(0) = \bar{F}^{-1}_{D-Q}(\beta^*)$$
$$\bar{m}_c(x) = i\mu_T + \frac{1-\alpha}{\alpha}\bar{F}_Q\left(\frac{b-x}{\alpha}\right)\left(\bar{F}_D\left(\frac{b-x}{\alpha}\right) + (v-1)\bar{F}_D\left(\frac{b-x}{\alpha} + x\right)\right)$$
$$\underline{m}_s(x) = (v-1)\bar{F}_{D-Q}(x)F_Q\left(\frac{b-x}{\alpha}\right)$$
$$\bar{m}(x) = \bar{m}_c(x) - \underline{m}_s(x)$$
$$x_- = \bar{m}^{-1}(0).$$

Assumption 1. *Demand and local supply are not positively correlated.*

Assumption 1 arguably holds in practice: for example, a severe event results in high demand and may partially destroy local supply or inhibit access to local supply (e.g., because of damage to infrastructure).

Proposition 2. (a) $x^* \le \bar{x}^* = \min\{(x_+)^+, b\}$. (b) *If* $b \ge \bar{b}$, *then* $x^* = \bar{x}^* = (\bar{F}^{-1}_{D-Q}(\beta^*))^+$. (c) *Suppose Assumption* 1 *holds. Then,* $x^* \ge \underline{x}^* = \min\{(x_-)^+, b\}$. (d) *Suppose Assumption* 1 *holds and* $b < \bar{b}$. *If* $\underline{x}^* = b$, *then* $x^* = b$. (e) $\underline{m}_c(x) \le m_c(x)$ *and* $\bar{m}_s(x) \ge m_s(x)$. *If Assumption* 1 *holds, then* $\bar{m}_c(x) \ge m_c(x)$ *and* $\underline{m}_s(x) \le m_s(x)$.

Proposition 2 shows that there is a threshold budget $\bar{b}$, above which the cost model exhibits a newsvendor-type trade-off. As noted, the value of β^* can be interpreted as the optimal unconstrained shortage probability. Its value is determined from three model primitives: cost per unit-period for holding prepo stock, i; average number of rapid-onset disasters per period or disaster frequency, $1/\mu_T$; and shortage cost expressed as the percentage increase in prepo purchase/transport cost or relative shortage cost $(v-c)/c = v-1$ (c normalized to 1). Because local supply is prioritized (and thereby, is used first to the extent that it is available and needed), the local supply cost multiple, α, plays no role in the optimal prepo level when $b \ge \bar{b}$. The value of x_+ is set to balance the unit cost of holding inventory, $i\mu_T$, against the marginal cost of a prepo stock shortage $(v-1)$ in the event that local supply is insufficient to cover demand that occurs with probability $P[D-Q > x_+]$.

4.2. Comparative Statics

Unless stated otherwise, we use increasing and decreasing in their weak sense throughout the paper.

Proposition 3. *Suppose* $b > \bar{b}$. (a) *Optimal prepo stock* x^* *is unaffected by changes in* (1) *local supply cost multiple* α, (2) *cash inflow rate* γ, (3) *mean and standard deviation of the emergency fund* μ_R *and* σ_R, *and* (4) *the volatility of time between disasters* σ_T. (b) *Optimal prepo stock* x^* *is increasing in* (1) *disaster frequency* $1/\mu_T$, (2) *shortage cost* v, (3) *initial budget* b, *and* (4) *average demand* μ_D. (c) *Optimal prepo stock* x^* *is decreasing in* (1) *holding cost rate* i, (2) *cost per unit of prepo stock* c, *and* (3) *average local supply* μ_Q. (d) *Optimal prepo stock* x^* *increases as* σ_D *and* σ_Q *increase (decrease) by the same proportion if* $x_+ > \mu_D - \mu_Q$ ($x_+ < \mu_D - \mu_Q$). (e) *If D and Q share the same marginal density function that is symmetric, then optimal prepo stock* x^* *increases as* (1) σ_D *and/or* σ_Q *increase (decrease) if* $x_+ > \mu_D - \mu_Q$ ($x_+ < \mu_D - \mu_Q$), *and* (2) *D and Q become more (less) negatively correlated if* $x_+ > \mu_D - \mu_Q$ ($x_+ < \mu_D - \mu_Q$).

The results in Proposition 3 offer lessons for how management may adjust prepo stock in response to changes in the environment. In our discussion of these results and lessons, we include parameter c (normalized to one) in relevant expressions because we address the effects of changes in prepo cost per unit. Some of the directional results follow from an understanding of the trade-off discussed (i.e., x_+ balances a downward pressure stemming from the cost of holding inventory $ic\mu_T$ against the upward pressure stemming from the relative shortage cost $v-c$) in conjunction with the prepo budget constraint (i.e., $x \le b/c$). From these observations, it follows rather naturally that increases in holding and purchase cost put downward pressure on prepo stock, whereas increases in disaster frequency, shortage cost, and funds put upward pressure on prepo stock. The directional effects related to moments and correlation of random variables are not directly tied to the trade-off and warrant discussion. First, changes in the volatility of time between disasters have no effect on prepo stock. Given $b > \bar{b}$, funding is sufficient to assure that local purchases will not be limited by the budget. As a consequence, the random local budget quantity does not enter into the prepo stock trade-off. Of course, optimal prepo stock is sensitive to changes in the first moment of T (μ_T) through its effect on holding cost. Second, as one may expect, optimal prepo increases as expected demand increases, and as expected, local supply decreases.

The impacts of changes in uncertainty and correlation are more subtle because of dependence on the shortage criticality of the item. We label a relief item as critical if the optimal unconstrained prepo stock (x_+) exceeds the expected mismatch between demand and local supply (i.e., $x_+ > \mu_D - \mu_Q$) and noncritical if the opposite inequality holds (i.e., $x_+ < \mu_D - \mu_Q$). (For example, for symmetric distributions of $D-Q$, the critical is a product with a newsvendor fractile more than 50%.) As the uncertainty in demand and local supply increase by the same proportion, the optimal prepo stock of critical relief items increases, and the optimal prepo stock of noncritical relief items decreases. A proportional increase in both σ_D and σ_Q increases the tails of the random mismatch $D-Q$ distribution, which decreases the shortage probability below β^* for a critical relief item, thus requiring an increase in prepo to compensate (and vice versa for a noncritical item). If the marginal distributions of D and Q are symmetric and identical, then the specific x_+ threshold for the reversal of the directional effect (i.e., $\mu_D - \mu_Q$) holds for an increase in σ_D and/or σ_Q (i.e., the threshold is not dependent on a proportional increase). The phenomenon that optimal prepo is increasing in demand and/or supply uncertainty for more critical items and decreasing for less critical items is likely to hold over a range of asymmetric distributions, although the specific threshold will depend on the specific distribution. The same behavior is associated with an increase in negative correlation between D and Q. This is because the introduction of negative correlation amplifies the variance of mismatch between demand and local supply (i.e., variance of $D-Q$), just as with increases in σ_D and/or σ_Q.

In addition to understanding whether prepo stock should increase or decrease in response to changes in the environment, the directional impacts may provide insight into the impact of candidate interventions by management. For example, it is clear that investments to improve forecasts of demand and/or local supply help reduce the costs of mismatch between demand and supply. However, other things being equal, such investments are likely to be more attractive for critical items than noncritical items. One relatively obvious reason for this is the high shortage cost of critical items compared with noncritical items (e.g., with no change in prepo stock, the value of improved forecasting is increasing in shortage cost). However, Proposition 3 tells us that optimal prepo stock of a critical relief item decreases as forecasts improve. Thus, in settings where the budget is limiting investment in prepo (i.e., $x_+ > b$), the rate of savings from improved forecasts is amplified for critical items, reflecting gains from both lower mismatch cost and lower cost of insufficient funding. In contrast, the inventory effect for noncritical relief items is reversed, possibly exacerbating the cost of budget limitations as forecast accuracy improves. Similarly, the positive effects of investments that increase average supply (i.e., reduce $\mu_D - \mu_Q$) or soften the degree of negative correlation between demand and supply are amplified for critical items relative to noncritical items.

Proposition 4. *Suppose* $b < \bar{b}$. (a) *Optimal prepo stock* x^* *can increase or decrease as the following parameters increase:* (1) *local supply cost multiple* α, (2) *average demand* μ_D, (3) *disaster frequency* $1/\mu_T$, (4) *uncertainty in demand* σ_D, (5) *uncertainty in supply* σ_Q, (6) *uncertainty in the emergency fund* σ_R, *and* (7) *uncertainty in time between disasters* σ_T. (b) *Optimal prepo stock* x^* *is increasing in* (1) *shortage cost* v, (2) *initial budget* b, (3) *cash inflow rate* γ, *and* (4) *average emergency fund* μ_R. (c) *Optimal prepo stock* x^* *is decreasing in* (1) *holding cost rate* i, (2) *cost per unit of prepo stock* c, *and* (3) *average local supply* μ_Q. (d) *If* $i = 0$, *then optimal prepo stock* x^* *is decreasing in disaster frequency* $1/\mu_T$.

As noted, the setting of Proposition 3 tends to fit HOs with a mission and resources to provide relief well beyond IRP or more generally, HOs for which local purchases during IRP are not constrained by the budget. In contrast, the setting of Proposition 4 tends to fit HOs that attend only to life-critical relief (e.g., exiting the region shortly after IRP) or more generally, where the budget limits what can be purchased locally.

We can observe that the system is more complex under the setting of Proposition 4. Recall that the setting of Proposition 3 is one where, at optimal prepo stock x^*, the local budget quantity constraint is never binding. As a consequence, optimal x^* balances the cost of holding prepo inventory against the shortage cost that occurs when demand is greater than the sum of local supply and prepo stock (to the extent that the prepo funding limit of b allows). The setting of Proposition 4 introduces an additional trade-off in the determination of x^*: balancing the cost of an excess local budget against the cost of an insufficient local budget. The cost of an excess local budget is associated with a global shortage and local budget more than local supply (i.e., $S(x) > x$ and $\hat{b}(x, R, T)/\alpha > Q$); cost would have been lower if prepo stock was higher. The cost of an insufficient local budget is associated with a shortage—either local or local and global—and local budget less than local supply (i.e., $S(x) > 0$ and/or $S(x) > x$ and $\hat{b}(x, R, T)/\alpha < Q$); cost would have been lower if prepo stock was lower (i.e., because it is more cost effective to cover shortages with local stock). In the following, we discuss results in Proposition 4 that differ from Proposition 3 and the intuition underlying the shifts in results.

Proposition 3 states that an increase in the frequency of disasters leads to an increase in prepo stock because this change reduces the cost of holding inventory. In the setting of Proposition 4, a greater disaster frequency also means less time to accrue funds before the next disaster. Because of increasing budget pressure, optimal prepo stock may decrease so as to open up more funds for local purchases. The greater the budget pressure, the more likely that optimal prepo stock will decrease as disaster frequency increases. Similar effect explains why optimal prepo stock is increasing in average demand under Proposition 3 but may be increasing or decreasing under Proposition 4. In particular, an increase in average demand increases budget pressure. As it is more cost effective to cover shortages using local supply, other things being equal, an increase in budget pressure translates to a relatively higher allocation of funds for local purchasing.

In contrast to Proposition 3, under the setting of Proposition 4, changes in α influence optimal prepo stock (because of the local budget constraint that can be binding). On one hand, an increase in α reduces the cost of prepo stock relative to local supply, which puts upward pressure on x^*. On the other hand, an increase in α reduces the local budget quantity, which puts downward pressure on x^*. Perhaps surprisingly, x^* is more likely to be increasing in α when local supply is inexpensive compared with prepo (i.e., small α) and less likely otherwise. The reason is that the percentage increase in local supply cost when α is increased by some fixed amount is larger when α is small (see the proof of Proposition 4).

Under the setting of Proposition 3, changes in uncertainty of the emergency fund or time between disasters do not affect optimal prepo stock x^*, and the directional effect of increasing demand and supply uncertainty depends on the item criticality. The setting of Proposition 4 introduces an additional trade-off that includes the probability of insufficient local budget on one side and the probability of a global shortage with an excess local budget on the other side. The directional effect of an increase in σ_R or in σ_T on these probabilities can go in either direction, depending on parameter values and probability distributions. Furthermore, the interaction between random mismatch $D - Q$ and random time between disasters, T, eliminates the possibility of a simple threshold to distinguish between whether x^* is increasing or decreasing in uncertainty.

Propositions 3 and 4 together offer insight into how a general trend of increasing frequency and severity of disasters (e.g., because of a combination of accelerating climate change and population growth) will affect HO supply management. If budget pressure is high, then management should decrease the investment in prepo stock as disaster frequency and severity increase. Overall, as disasters become more frequent and severe with a consequent increase in budget pressure, we can expect HOs to place greater emphasis on local supply during the immediate relief period.

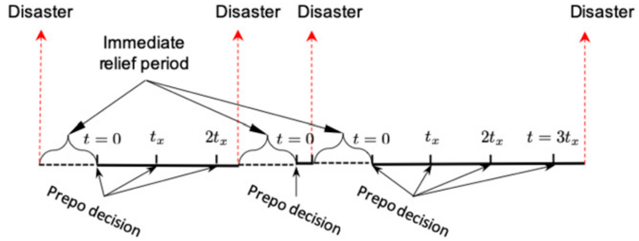
5. Dynamic Prepo Policy

In this section, we relax our assumption that the prepo decision at the end of an immediate relief period is fixed until the next disaster occurs. We define and analyze a model wherein the prepo level is reviewed on a periodic basis.

Let t_x denote the length of the time between prepo decisions (e.g., if t_x periods have elapsed since the last review (and no disaster has occurred in the interim), then prepo is reoptimized). We refer to t_x as the *prepo review period*. As in Section 3, we define a model of expected cost over a prepo cost cycle. The timing details of a dynamic prepo policy are illustrated in Figure 2. The figure illustrates three prepo cost cycles, with prepo updated twice during the first cycle, not updated during the second cycle, and updated three times during the third cycle.

The notation of our dynamic model follows the notation of Section 3 with the following adjustments and additions. The prepo decision at the beginning of period n is x_n. Suppose that a disaster occurs in period n (i.e., $T \in [(n-1)t_x, nt_x)$). Let T_n denote the random time that the disaster occurs within period n (i.e., $T_n = (T - (n-1)t_x) | T \in [(n-1)t_x, nt_x)$ and $\mu_n = E[T_n] = E[T - (n-1)t_x | (n-1)t_x \le T < nt_x]$). Recall that b is the initial budget (at time zero before prepo is updated) that includes the carryover prepo stock from the last cycle. We similarly let b_n denote the

Figure 2. (Color online) Illustration of Prepo Cost Cycles in a Dynamic Setting



budget at the beginning of period n that includes prepo stock determined in the previous period. For example, if $n = 1$, then $b_1 = b$. If $n > 1$, then b_n includes the cash inflow between the start of period 1 (time zero) and the start of period n. Thus, the budget available for prepo at the beginning of period n given that a disaster has not yet occurred is $b_n = b + \gamma(n-1)t_x$.

Suppose that at the beginning of period n, a disaster has not yet occurred. The decision maker has the opportunity to update prepo. In the following, we define relevant functions and the decision maker's problem at the beginning of period n. The functions include prepo as an argument that we denote as x.

If a disaster occurs in period n at time T_n, the random budget for local spend as a function of prepo at the start of period n, emergency fund, and disaster time is

$$\hat{b}_n(x, R, T_n) = b + \gamma((n-1)t_x + T_n) + R - x$$
$$= b_n + \gamma T_n + R - x.$$

Let $x_1, x_2, \ldots, x_{n-1}$ denote the prepo decisions in the cycle prior to period n. Then, the sum of past prepo decisions since the beginning of the prepo cost cycle is y_n: that is,

$$y_n = \sum_{k=1}^{n-1} x_k, \text{ with } y_1 = 0.$$

We add a subscript to the random local shortage function, $S_n(x)$ (i.e., random local shortage given that a disaster occurs in period n with prepo x):

$$S_n(x) = \left(D - \min\left\{Q, \frac{\hat{b}_n(x, R, T_n)}{\alpha}\right\}\right)^+$$
$$= \left(D - \min\left\{Q, \frac{b_n + \gamma T_n + R - x}{\alpha}\right\}\right)^+.$$

With this notation, the prepo decision problem at the beginning of period n (given that a disaster has not yet occurred) is

$$C_n^*(y_n) = \min_{x \le b_n} \left\{ \begin{array}{c} \bar{C}_n(x, y_n) P[T \in [(n-1)t_x, nt_x) \,|\, T \\ \ge (n-1)t_x] + C_{n+1}^*(x + y_n) \\ \times P[T \ge nt_x | T \ge (n-1)t_x] \end{array} \right\}, \quad (6)$$

where $\bar{C}_n$ is the expected cost given that the disaster occurs in period n,

$$\bar{C}_n(x, y_n) = it_x y_n + i\mu_n x + (1-\alpha)E[S_n(x)] + (v-1)E[(S_n(x)-x)^+],$$

and $C_n^*(y_n)$ is the optimal expected cost to go given y_n and the disaster occurs in period n or later. Proposition 5 characterizes the optimal decision and identifies lower and upper bounds on optimal prepo. It relies on the following notation that largely follows the notation in Section 4, although it is augmented to account for period n. Let Ω denote the support of (D, Q, R, T_n). Define $\Omega_i(x)$ as in Section 4.1 with b replaced by b_n, $i = 1, 2, 3$. Let

$$p_n = P[T \in [(n-1)t_x, nt_x) | T \geq (n-1)t_x]$$
$$m_c(x) = i\left(\mu_n + t_x \frac{1-p_n}{p_n}\right) + \frac{1-\alpha}{\alpha}(P[\Omega_1(x)] + (v-1)P[\Omega_2(x)])$$
$$\underline{m}_c(x) = i\left(\mu_n + t_x \frac{1-p_n}{p_n}\right)$$
$$\beta_n^* = \frac{i\left(\mu_n + t_x \frac{1-p_n}{p_n}\right)}{v-1}$$
$$\bar{m}_c(x) = i\left(\mu_n + t_x \frac{1-p_n}{p_n}\right) + \frac{1-\alpha}{\alpha}\bar{F}_Q\left(\frac{b_n - x}{\alpha}\right) \times \left(\bar{F}_D\left(\frac{b_n - x}{\alpha}\right) + (v-1)\bar{F}_D\left(\frac{b_n - x}{\alpha} + x\right)\right)$$
$$\bar{b}_n = \max\{\alpha \min\{d, q\} - r : (d, q, r) \in \Omega\} - \gamma \underline{T}_n + (x_+)^+.$$

(Following the notation convention in Table 1, the support of T_n is $[\underline{T}_n, \bar{T}_n]$).[2] Define all other notations similarly as in Section 4.1 with b replaced by b_n and β replaced by β_n.

Proposition 5.

a. *The optimal prepo decision at the beginning of period n is*

$$x_n^* = \min\{(x^o)^+, b_n\}. \tag{7}$$

Furthermore, $x_n^* \leq \bar{x}_n^* = \min\{(x_+)^+, b_n\}$.

b. *If $b_n \geq \bar{b}_n$, then $x_n^* = \bar{x}_n^* = (\bar{F}_{D-Q}^{-1}(\beta^*))^+$.*

c. *Suppose Assumption 1 holds. Then, $x_n^* \geq \underline{x}_n^* = \min\{(x_-)^+, b_n\}$.*

d. *Suppose Assumption 1 holds and $b_n < \bar{b}_n$. If $\underline{x}_n^* = b_n$, then $x_n^* = b_n$.*

e. *$\underline{m}_c(x) \leq m_c(x)$ and $\bar{m}_s(x) \geq m_s(x)$. If Assumption 1 holds, then $\bar{m}_c(x) \geq m_c(x)$ and $\underline{m}_s(x) \leq m_s(x)$.*

We note that a key idea underlying Proposition 5 is that we are able to decompose (6) into a collection of terms that depend on y_n and x and a term that does not (see the proof of Proposition 5). Although the optimal cost-to-go function cannot be computed (because the last term is unknown), the key insight is that terms that depend on y_n and x can be expressed, thereby allowing optimal prepo to be characterized.

We see that the results in Proposition 5 are similar to the results in Propositions 1 and 2 that apply to the static prepo model. In order to discuss differences, we begin with Table 2, which summarizes the model elements that differ between the static and dynamic models.

The results in Table 2 highlight drivers of differences in optimal prepo between static and dynamic models. Differences can be traced to three effects: (1) prepo budget effect, (2) local budget effect, and (3) marginal holding effect.

5.1. Prepo Budget Effect

For the static model, there is a single prepo decision subject to budget limit b. The dynamic model updates prepo each period until a disaster occurs. The initial budget is identical to the static model but increases thereafter if $\gamma > 0$ (i.e., $b = b_1$ and $b_n < b_{n+1} \forall n$). Note that there is no prepo budget effect if $\gamma = 0$.

5.2. Local Budget Effect

The marginal cost and savings functions include the probabilities of events defined through sets Ω_1, Ω_2, and Ω_3, which are affected by the random local budget: that is,

$$P[\Omega_1(x)] = P\left[D > \frac{\hat{b}_\bullet}{\alpha}, Q > \frac{\hat{b}_\bullet}{\alpha}\right]$$
$$P[\Omega_2(x)] = P\left[D > \frac{\hat{b}_\bullet}{\alpha} + x, Q > \frac{\hat{b}_\bullet}{\alpha}\right]$$
$$P[\Omega_3(x)] = P\left[D > Q + x, Q \leq \frac{\hat{b}_\bullet}{\alpha}\right],$$

where $\hat{b}_\bullet = \hat{b}(x, R, T)$ for the static model and $\hat{b}_\bullet = \hat{b}_n(x, R, T_n)$ for the dynamic model. Note that there is no local budget effect if $\gamma = 0$ (i.e., $\hat{b}(x, R, T) = \hat{b}_n(x, R, T_n)$ for any n).

Table 2. Key Differences in Static and Dynamic Model Elements

Static model	Dynamic model
Prepo quantity constraint because of budget	
$x \leq b$	$x \leq b_n = b + \gamma(n-1)t_x$
Local quantity constraint because of budget	
$\hat{b}(x, R, T) = b + \gamma T + R - x$	$\hat{b}_n(x, R, T) = b_n + \gamma T_n + R - x$
$x_L \leq \frac{\hat{b}(x,R,T)}{\alpha}$	$x_L \leq \frac{\hat{b}_n(x,R,T_n)}{\alpha}$
Marginal prepo holding cost	
$i\mu_T$	$i(\mu_n + t_x \frac{1-p_n}{p_n})$

5.3. Marginal Holding Effect

The marginal cost functions include the marginal holding cost of prepo. As shown in Table 2, the marginal holding cost under the static model is $i\mu_T$, and the marginal holding cost under the dynamic model is $i(\mu_n + t_x(1-p_n)/p_n)$. The following lemma presents an identity that is useful comparing the marginal holding costs. Recall that μ_n is the expected time within prepo review period n that a disaster occurs (given that it occurs in period n) as of the beginning of the period. Let μ_n^+ denote the expected remaining time to disaster as of the beginning of prepo review period n, that is,

$$\mu_n^+ = E[T-(n-1)t_x | T-(n-1)t_x \geq 0].$$

Lemma 1. *For any probability distribution of random time to disaster T,*

$$\mu_n + t_x\frac{1-p_n}{p_n} = \mu_n^+ + \left(\mu_n^+ - \mu_{n+1}^+\right)\frac{1-p_n}{p_n}.$$

In the remainder of this section, we focus on the case where T is an exponential random variable. (In Section 6, we provide justification for why the exponential distribution is often a reasonable assumption in practice.). However, Lemma 1 can be useful for clarifying the marginal holding effect for different probability distributions of time to disaster. To illustrate this point, we present a brief example where T is uniformly distributed between zero and Nt_x periods. For convenience, we assume that N is even. Then,

$$\mu_n^+ = t_x\frac{N-(n-1)}{2}$$

$$\mu_n^+ - \mu_{n+1}^+ = \frac{t_x}{2}$$

$$\frac{1-p_n}{p_n} = \frac{(N-(n-1)-1)/(N-(n-1))}{1/(N-(n-1))} = N-n$$

$$\mu_n^+ + \left(\mu_n^+ - \mu_{n+1}^+\right)\frac{1-p_n}{p_n} = t_x\left(\frac{N-(n-1)}{2} + \frac{N-n}{2}\right) = t_x\left(\frac{1}{2} + N - n\right)$$

$$\mu_T = \mu_1^+ = \frac{t_xN}{2}$$

$$\begin{aligned} t_x\left(\frac{1}{2}+N-1\right) &= \mu_1^+ + \left(\mu_1^+ - \mu_2^+\right)\frac{1-p_1}{p_1} \\ &> \mu_2^+ + \left(\mu_2^+ - \mu_3^+\right)\frac{1-p_2}{p_2} > \dots \\ &> \mu_{N/2}^+ + \left(\mu_{N/2}^+ - \mu_{N/2+1}^+\right)\frac{1-p_{N/2}}{p_{N/2}} \\ &= \mu_T = \frac{t_xN}{2} > \dots \\ &> \mu_N^+ + \left(\mu_N^+ - \mu_{N+1}^+\right)\frac{1-p_N}{p_N} = \mu_N^+ = \frac{t_x}{2}. \end{aligned}$$

For uniform time to disaster, the marginal cost of holding prepo is initially higher in the dynamic model, matches the static model at the midpoint of the maximum time to disaster, and is smaller thereafter. Thus, if $\gamma = 0$ (so budget does not grow over time), then the optimal prepo values under uniformly distributed time to disaster satisfy

$$\begin{aligned} x_1^* \leq x_2^* \leq \cdots \leq x_{N/2-1}^* \leq x_{N/2}^* \\ = x^* \leq x_{N/2+1}^* \leq \cdots \leq x_N^*. \end{aligned} \quad (8)$$

Furthermore, if the upper-bound constraint on prepo is not binding, then the inequalities are strict. Equation (8) can be interpreted as a pure manifestation of the marginal holding effect under a uniform distribution (i.e., behavior when prepo and local budget effects are removed). Our next result, which is a corollary to Lemma 1, shows that the marginal holding effect disappears when time to disaster is an exponential random variable. This result underlies Proposition 6 that provides a comparative characterization of prepo decisions for static versus dynamic models and Proposition 7 that characterizes the effect of increasing t_x.

Corollary 1. *If T is an exponential random variable, then*

$$\mu_n + t_x\frac{1-p_n}{p_n} = \mu_T.$$

Proposition 6. *Suppose that T is an exponential random variable.*

a. $x_1^* \leq x^*$ *and* $x_n^* \leq x_{n+1}^*$ *for all n.*
b. *If* $\gamma = 0$, *then* $x^* = x_n^*$ *for all n.*
c. *If* $b \geq \bar{b}$, *then* $x^* = x_n^*$ *for all n.*
d. *Suppose Assumption 1 holds and* $\gamma > 0$. *If* $\underline{x}^* = b$, *then* $x^* = x_1^*$ *and* $x^* \leq x_n^*$ *for all n.*

Proposition 7. *Suppose that T is an exponential random variable. Then,* x_1^* *is increasing in the length of the review period* t_x.

Let us recap our main conclusions from analysis given exponentially distributed time to disaster. If donations received over time either are not allocated to the immediate relief period or are nonexistent (i.e., $\gamma = 0$) or if the HO has enough cash reserves to assure that the local budget constraint is nonbinding (e.g., $b \geq \bar{b}$), then the *static and dynamic models are equivalent*. In these cases, it is optimal to set prepo at the beginning of the prepo cost cycle and leave it fixed until the next disaster occurs. On the other hand, if $\gamma > 0$, then dynamic optimal prepo is nondecreasing over time until the disaster occurs.

Proposition 6(a) compares optimal prepo over the extremes of an infinite review period (static model) and a finite review period (dynamic model). For a given budget, we see that dynamic optimal prepo is dominated by static optimal prepo (i.e., $x_1^* \leq x^*$). For this comparison, the prepo budget effect and marginal holding effect are not present (i.e., because of the same budget and exponential time to disaster). We see that recourse value of the dynamic model is solely driven by the local budget effect. In particular, the option to update prepo at the end of the review period in the event that a disaster does not occur adds value by providing additional funds for local spend that the event that a disaster does occur. Proposition 7 shows that this phenomenon holds outside of the extremes of finite versus infinite review period (i.e., that optimal prepo is nondecreasing in review period length). Key drivers of the significance of this effect are the funds' inflow rate and the budget. The length of the review period has no effect on optimal prepo if $\gamma = 0$ (Proposition 6(b)), if budget is high (Proposition 6(c)), or if budget is low (e.g., a binding constraint on prepo). For our calibrations in the next section, we find that for the same budget, there is little or no difference in optimal prepo for dynamic and static models. Although there is minimal difference in the time zero prepo values (e.g., the local budget effect is largely insignificant in our calibrations), we do see differences as time elapses (e.g., given $\gamma > 0$, dynamic optimal prepo increases over time, reflecting the prepo budget effect).

Propositions 3 and 4 show how the optimal static prepo changes as various parameters increase. How do these comparative statics change for the dynamic model? In other words, what are the effects of increases in parameters at the beginning of period n on optimal prepo? By examining Table 2, it is clear that all of the directional effects identified in Propositions 3 and 4 continue to hold for the dynamic model (i.e., b_n, $\bar{b}_n$, and x_n^* take the place of b, $\bar{b}$, and x^* in the propositions).

Note that the dynamic model reduces to the static model when the prepo review period is infinite. In the next section, we present numerical results that illustrate the effects of changes in various model parameters on prepo decisions, including the length of the prepo review period.

6. Numerical Illustrations

Through numerical analysis in this section, we seek to provide insight into (i) settings where optimal prepo is likely to be closer to the lower bound, closer to the upper bound, or near the middle; (ii) conditions under which expected cost is relatively insensitive to different levels of prepo between the lower and upper bounds; and (iii) the value of emergency fund. Such insights can be useful for understanding how alternative assumptions on costs and uncertainty translate into reasonable prepo targets and ultimately, for helping to make prepo decisions that take advantage of (likely limited) available information.

6.1. Setting

We designed our numerical experiments *inspired* by actual response operations of a large international HO (hereafter, focal humanitarian organization (FHO)) that owns a warehouse in Lipa city in the Philippines. We assume that FHO's Philippines warehouse covers emergency demands if a rapid-onset disaster hits Indonesia, Guinea, the Philippines, Myanmar, Bangladesh, Vietnam, Cambodia, and Laos. Using multiple sources such as the Emergency Events Database[3] and Glide Number,[4] we collected disasters data from June 1, 2006 to June 31, 2018. Our data contain information about the date and type of each disaster and the number of affected people. We excluded disasters with transitional period (e.g., storm, tropical cyclone, and disasters that affected less than 1,000 people). We counted 66 rapid-onset disasters that affected one of the mentioned countries. To estimate the frequency of disaster, we found that the average time between two consecutive disasters is 67.5 days, with standard deviation of 65.3 days. Therefore, we assume events occur at an average rate of about six per year, and FHO cannot discount the possibility of immediate relief activities for multiple events occurring nearly simultaneously (i.e., in effect, $\underline{T} = 0$). Without information to indicate otherwise, we estimate that the time between relief events is an exponential random variable.[5]

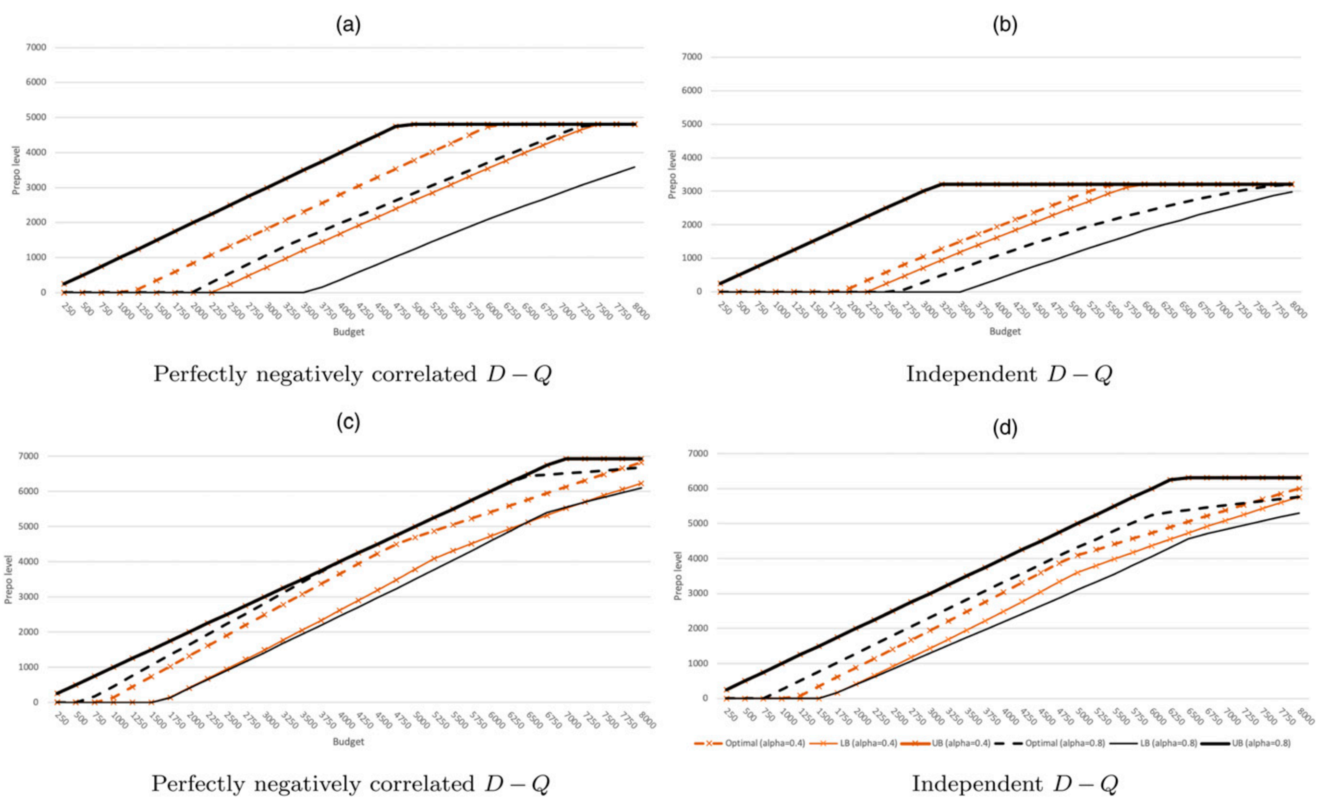
Over the course of 12 years, we found seven disasters that affected more than 700,000 people. Because these disasters typically attract the attention of many large international HOs, it is understandable that FHO sets a demand cap to cover. The median number of affected is around 50,034. Therefore, in our numerical study, we let the demand for a relief item range from 50,000 to 700,000 beneficiaries. Assuming an average total landed cost of \$50 (approximate landed cost of a kitchen kit with original price of \$35), we convert demand in kits to demand in 1,000 prepo dollars and assume that demand is uniformly distributed over this range (i.e., $D \sim U[500, 7000]$). In order to illustrate the effect of highly critical versus less critical items, we consider two rather extreme values $v \in \{1.2, 7\}$. We use the uniform distribution for supply. It seems reasonable for the low end to be zero (e.g., disaster completely destroys local supply, or the product is simply not available in the area). Typically, the number affected may be a relatively small fraction of the population in the area of the disaster. If fraction is small enough, then there may be enough to cover all affected. Based on this, the upper limit of supply at 95% of the maximum demand of 7000 (i.e., $Q \sim U(0, 6650)$). We consider two rather extreme values for local supply cost, $\alpha \in \{0.4, 0.8\}$. We set a base level of emergency fund at a level sufficient to cover 10% of demand and increased this by absolute of one-third and two-thirds (i.e., $R \in \{0.1\alpha D, 0.43\alpha D, 0.76\alpha D\}$). We use stochastic optimization with 100,000 trials per simulation via the Analytic Solver Platform from Frontline Systems to generate the results.

Given the space limitation, we only illustrate a few plots, although the insights hold based on over 6,000 numerical experiments. Figure 3 reports UB, LB, and optimal prepo for cases of highly critical and less critical items per cycle for both independent demand/supply (panels (b) and (d)) and demand/supply with perfect negative correlation (panels (a) and (c)) as budget varies. Figure 5 reports optimal prepo and optimal average costs for both highly critical and less critical items when the emergency fund changes.

We offer four observations. First, the basic patterns in prepo curves and expected cost curves are relatively stable across changes in correlation, changes in shortage cost rate, and changes in local supply cost ratio. We see diminishing return to increases in budget and higher marginal value of budget for the high shortage cost item, both of which are to be expected. In general, optimal prepo and bounds are larger when (i) demand and supply are negatively correlated, which motivates more investment on prepo for regions with limited supply, and (ii) the item is more critical.

Second, Figure 3 shows when budget is above the threshold ($b > \bar{b}$), optimal prepo (and the upper bound) is the same for $\alpha = 0.4$ and $\alpha = 0.8$, aligned with the results in Propositions 2 and 3. However, when the budget is low, optimal prepo is *decreasing* in α for the less critical item, whereas it is *increasing* for the more critical item. This is because of the additional dimension in the trade-off that arises when budget is below the threshold (as discussed after Proposition 4; i.e., balancing the cost of an excess local budget against the cost of an insufficient local budget). In particular, an increase in α increases budget pressure (fewer units in total can be purchased), which puts downward pressure on optimal prepo. On the other hand, an increase in α reduces the savings from substituting spend on prepo with spend on local supply, which puts upward pressure on optimal prepo. At $v = 1.2$, the substitution savings play a relatively large role (because the consequence of a global shortage is relatively less), resulting in a decrease

Figure 3. (Color online) Panels (a) and (b) Show the Optimal Prepo x^* and Lower and Upper Bounds ($\underline{x}^*$ and $\bar{x}^*$) When $v = 1.2$, and Panels (c) and (d) Show the Same Values When $v = 7$



Notes. Other parameters: $\alpha \in \{0.4, 0.8\}$, $i = 0.2$, $\mu_T = 1/6$, $b \in [250, 8000]$, $\gamma = 500$, $D \sim U(500, 7000)$, and $Q \sim U(0, 6650)$. Emergency fund is sufficient to cover 10% of demand, and budget threshold, $\bar{b}$, equals 6,083 for panel (a), 5,584 for panel (b), 8,201 for panel (c), and 8,687 for panel (d). (a and c) Perfectly negatively correlated $D - Q$. (b and d) Independent $D - Q$.

Figure 4. Approximate Solution: Optimal Prepo Is Closer to Upper Bound or Lower Bound Depending on Local Price, Item Shortage Cost, and Local Supply Reliability

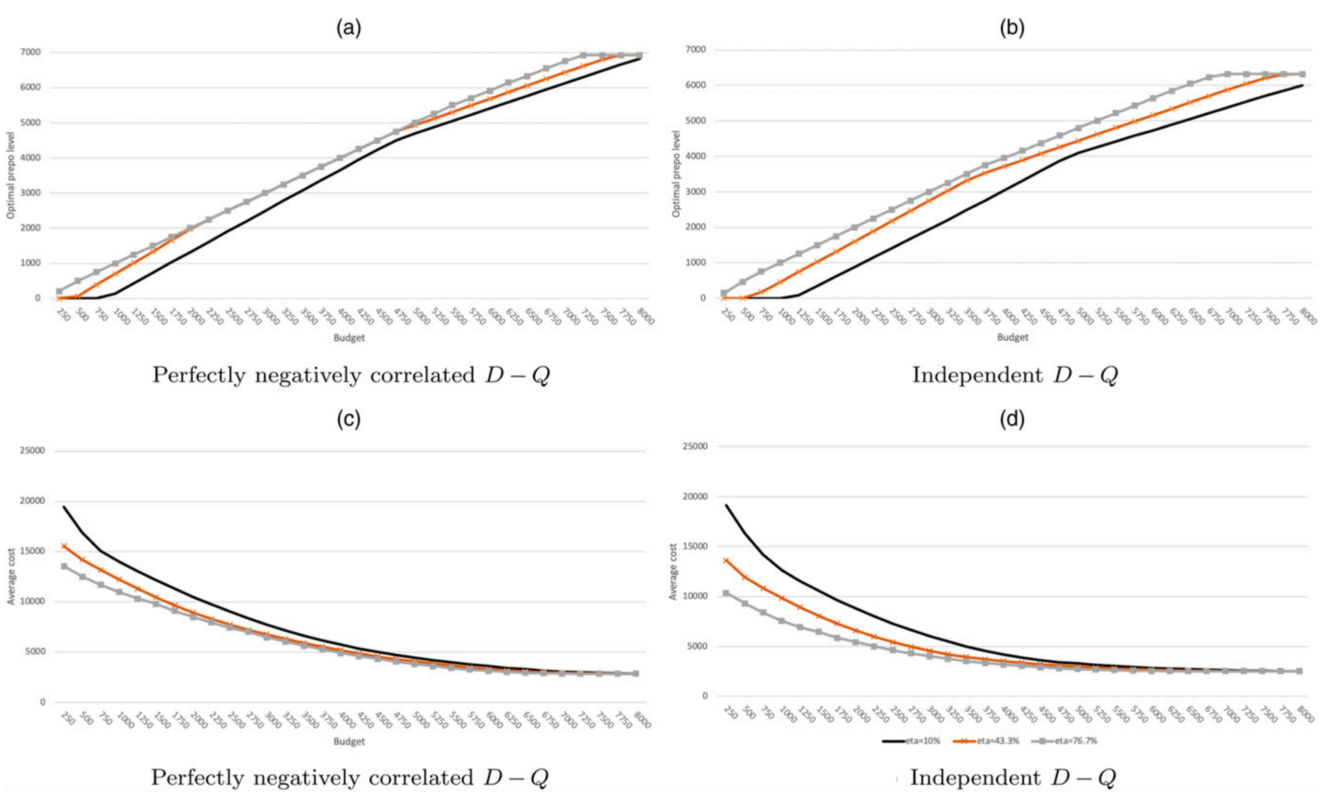
	High local price		**Low local price**	
	High shortage cost	Low shortage cost	High shortage cost	Low shortage cost
Low emergency fund	Close to UB	Close to UB if D-Q **independent**	Close to LB if D-Q independent but close to UB if correlated	
High emergency fund	Close to UB			

in x^* as α increases. As v increases, budget pressure becomes relatively stronger, causing the directional effect of increasing α on x^* to reverse at $v = 7$. The overarching insight is that the behavior of the system becomes much more complex and nuanced because of the added dimension in the prepo cost trade-offs under a limited budget.

Third, the curves provide some insight into the relative accuracy of the UB versus the LB. When budget is small (e.g., below the threshold), we see that optimal prepo lies between the UB and LB. However, optimal prepo tends to be closer to the LB for independent $Q - D$ and closer to the UB for correlated $Q - D$. The LB uses simplified expression for marginal cost and marginal savings that exploits bounds on probability expressions that are exact when D and Q are independent. Thus, when budget is low, the LB gap becomes relatively tighter when D and Q are independent, compared with negatively correlated. Figure 4 provides a summary of approximate solutions with respect to changes in local price, shortage cost, and emergency fund.

Fourth, as shown in Figure 5, the value of emergency funds is somewhat similar for independent and

Figure 5. (Color online) Panels (a) and (b) Show the Optimal Prepo x^*, and Panels (c) and (d) Show the Corresponding Average Cost for Critical Item ($v = 7$) and When Emergency Fund Varies from 10% to 43.3% and 76.7%



Notes. Other parameter values are $i = 0.2$, $\mu_T = 1/6$, $b \in [250, 8000]$, $\gamma = 500$, $\alpha = 0.4$, $D \sim U(500, 7000)$, and $Q \sim U(0, 6650)$. (a and c) Perfectly negatively correlated $D - Q$. (b and d) Independent $D - Q$.

correlated cases, with directional effects as expected (e.g., as emergency funds increase, prepo increases, and expected cost decreases). However, the effects disappear when budget is high (i.e., when budget is above the threshold, an increase in emergency funds has no effect on prepo or cost). Our experiments show that emergency funds generate more value when demand/supply is independent, and shortage cost is very high.

Figure 5 triggers a question of whether (or when) an increase in emergency fund is more or less valuable than an increase in initial budget. On one hand, an additional dollar (on average) in the emergency fund can potentially be more valuable because it aligns with demand (e.g., donations are high when demand is high). On the other hand, an additional dollar in the initial budget is more flexible—it can be used for prepo or local supply. Figure 6 shows the value of an increase of $500 added to the initial budget versus $500 average added to the emergency fund at different budget levels. The figure illustrates diminishing return to increases in the emergency fund, as well as to increases in budget (as illustrated in this figure as well as Figure 5). Whether an increase in the emergency fund adds more or less value than a similar increase in the budget depends on parameter values; neither funding source dominates the other. However, the value of an increase in the emergency fund decreases as negative correlation increases. This is because of greater misalignment between the amount of emergency funds available and the amount of local supply (e.g., when demand is high, donations tend to be high, but local supply tends to be low). Our results suggest that the allocation of efforts to raise funds from different sources will likely require a balanced strategy to achieve the greatest value.

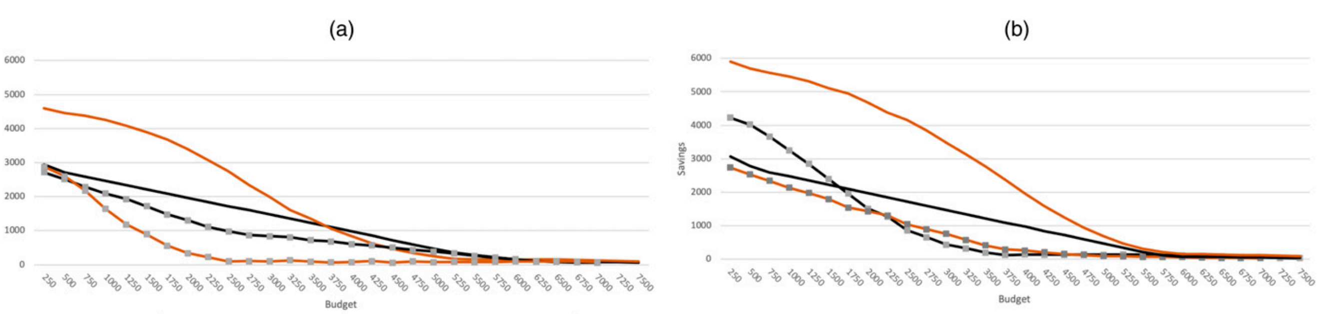
Panels (a) and (b) of Figure 7 show optimal prepo for review periods of one month, six months, and static optimal prepo (e.g., $t_x = \infty$). Panels (c) and (d) of Figure 7 show the change optimal prepo as the review period increases from one to six months and from one month to infinity (i.e., static model). From Proposition 7, we know that optimal prepo is increasing (in the weak sense) in the review period length. However, across all parameter combinations in our experiments, we find that changes in the review period length have a relatively small effect on optimal prepo, as illustrated in the figure. There is no difference in optimal prepo as the length of the review period increases when the budget is either very small or very large. Differences do arise at moderate budget levels but are generally small relative to optimal prepo.

7. Extension: Prepo Prioritized over Reactive Stock

For some settings and relief items, the local price may be higher than prepo. Our preceding analysis assumes that reactive stock is less expensive than prepo (i.e., $\alpha \leq 1$). In this section, we summarize how our main conclusions change if the inequality is reversed. We refer the interested reader to Eftekhar and Webster (2020) for additional detail.

The analysis of our model with $\alpha < 1$ leads to two main conclusions on the structure of the prepo optimization problem and two main results that are consequences of this structure. The two main structural conclusions are (1) that the cost function is convex in prepo and (2) that there is a threshold budget that delineates a structural change in the prepo optimization problem. The budget threshold corresponds to the value at which the constraint on local spend is assured to be nonbinding. Above the threshold, the structure is such that the optimization problem exhibits a classic newsvendor-type trade-off. Optimal prepo is determined from the fractile of a single random variable (i.e., the difference between random demand and random local supply). When the budget is below the threshold, an additional trade-off

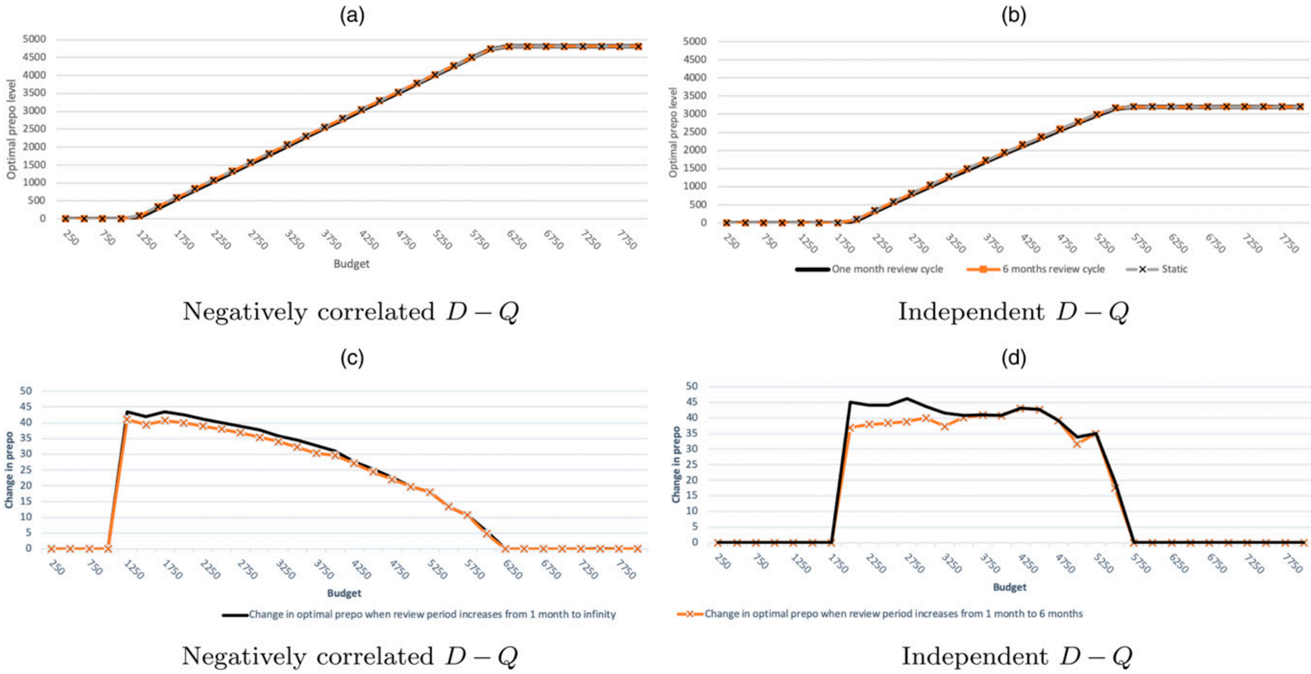
Figure 6. (Color online) Plots Show the Savings of Adding Emergency Fund Vs. Increasing the Initial Budget



Negatively correlated $D-Q$ Independent $D-Q$

Notes. Parameter values are $\alpha = 0.8$, $v = 7$, $i = 0.2$, $\mu_T = 1/6$, $b \in [250, 8000]$, $\gamma = 500$, $D \sim U(500, 7000)$, and $Q \sim U(0, 6650)$. (a) Negatively correlated $D-Q$. (b) Independent $D-Q$.

Figure 7. (Color online) Panels (a) and (b) Show Prepo Levels for Different Review Cycles, and Panels (c) and (d) Compare the Difference Between Prepo Levels When Review Cycle Increases



Notes. Parameter values are $\alpha = 0.4$, $v = 1.2$, $i = 0.2$, $\mu_T = 1/6$, $b \in [250, 8000]$, $\gamma = 500$, $D \sim U(500, 7000)$, and $Q \sim U(0, 6650)$, and emergency fund is sufficient to cover 10%. (a and c) Negatively correlated $D - Q$. (b and d) Independent $D - Q$.

arises in the determination of optimal prepo: the cost of excess local budget versus the cost of insufficient local budget. The effect is to decrease the marginal value of prepo, which translates to a reduction in optimal prepo as budget is reduced. An understanding of this threshold and the corresponding change in the drivers of marginal value of prepo underlie two main results. First, the monotonic marginal value function allows us to identify upper and lower bounds on optimal prepo that are easy to compute and interpret. Second, an understanding of the (well-behaved) marginal value function provides the intuition into the directional effects of changes in parameters on optimal prepo.

How are these main conclusions and consequent results affected by $\alpha > 1$? The two main conclusions continue to hold. The cost function continues to be convex in prepo. However, the proof of this result requires more nuanced arguments and hints at additional complexity. There is a threshold budget that delineates a structural change in the prepo optimization problem, and as for $\alpha < 1$, this threshold corresponds to the value at which the constraint on local spend is assured to be nonbinding.

As we look deeper, we see structural differences. When budget is above the threshold, the optimization problem does not conform to a newsvendor-type structure for which the optimal solution is determined by the fractile of a random variable. In particular, optimal prepo is obtained from an identity containing fractiles of two random variables: (1) random demand and (2) difference between random demand and random local supply. However, the key structural difference arises when budget drops below the threshold. In particular, the marginal value of prepo is monotonic (increasing) in budget when $\alpha < 1$ and is nonmonotonic when $\alpha > 1$. This difference in the marginal value function affects the behavior of the system in two ways that contrast starkly with the case of $\alpha < 1$. First, the model is no longer amenable to simple upper and lower bounds (i.e., the bounds from $\alpha < 1$ are no longer valid). Second, the nonmonotonic marginal value function leads to some directional effects of changes in parameters on optimal prepo that are opaque and surprising, the most salient of which are the impact of an increase in budget and an increase in prepo cost. Intuition might suggest that prepo will increase in budget (as is the case of $\alpha < 1$), especially because it is prioritized over reactive stock. However, increases in budget may lead to decreases in optimal prepo (a phenomenon that explains why the upper bound of $\alpha < 1$ does not translate to $\alpha > 1$). Similarly, it is possible that investment in prepo will increase as prepo becomes more expensive. These phenomena are more likely to arise when local supply is relatively plentiful and the cost of reactive stock is relatively small compared with shortage cost. The intuition for this result is that the savings from additional prepo can drop precipitously as budget increases (or the cost of prepo decreases) reduce the likelihood of a global shortage.

8. Summary and Conclusion

Through extensive interactions with HOs and their executives, we identified several important features in HOs' practices and challenges that have not been explored in the operations management literature on relief item supply management. These features include the distinction between local supply (reactive stock) and central supply (prepo stock); the high transportation cost of prepo stock (that makes it more expensive than the reactive stock); the importance and priority of reactive stock, a budget constraint, and the uncertainty of demand and local supply; and the inflows of funds such as donations during each decision cycle and at the onset of a disaster event. Although practitioners must constantly cope with these complexities, there is no clear guideline on how to act. To fill this gap between theory and practice, in this paper we develop an analytical framework that explicitly takes into account these new features. We obtain closed form solutions and efficient algorithms to determine the optimal prepositioned stock level in anticipation of the next disaster event, with consideration of uncertainty of time to the next event and the associated holding cost, uncertainty of demand, priority of using (uncertain) local supply, and uncertainty of budget. We consider both a static model, in which we make the prepo decision only at the beginning of the decision cycle, and a dynamic model, in which we can periodically purchase prepo before the next disaster event. We also derive and discuss extensive comparative statics analysis to reveal insights. Finally, we conduct an intensive numerical experiment to illustrate various effects. The design of the numerical experiment is inspired and resembles the real response operations of a large international HO.

The main lessons from our study can be summarized as follows. First, we provide simple and easy to implement methods (e.g., lower and upper bounds computed using Excel) that a manager may use to identify ranges of reasonable prepo values under differing assumptions of cost rates and probability distributions of demand, local supply, and time between disasters. Additionally, our comparative statics results (Propositions 3 and 4) help a manager to gauge the directional effects of changes in parameter estimates. As a related but more speculative finding, our numerical results hint that it is less costly to error on the side of too little prepo than too much prepo. In particular, we find that even when "true" optimal prepo is midway between lower and upper bounds, the cost at the lower bound is closer to the optimal cost (e.g., cost function is flatter to the left of optimal than to the right). In the face of limited data, a manager may wish to favor prepo values on the left side of the range of plausible values. In practice, it is often difficult to obtain accurate estimates of parameters. Our methods and results can help a manager identify a prepo target that takes a reasonable middle ground when data for estimating parameters and probability distributions are limited.

Second, initiatives to improve demand forecast accuracy and/or reduce the negative correlation between demand and local supply serve to shrink the mismatch cost between demand and supply. The higher the shortage cost rate, the larger the benefit from such investment. This conclusion is not surprising. However, we identify an interaction between such initiatives and the level of optimal prepo stock that can amplify the value for critical items and attenuate the value for noncritical items (i.e., because of loosening versus a tightening of the budget constraint). The main lesson is that a focus on mismatch cost may understate the value of such initiatives for critical items.

Third, our models allow a manager to estimate the value of relaxing a binding constraint on local spend. Large HOs may have sufficient reserves (or access to a line of credit) that can be spent on local supply during the immediate relief period with assurance that these reserves will be replenished through donations during the event. We know from Proposition 2 that the complex four-dimensional trade-off that arises in the presence of a binding local spend constraint dissolves into a relatively simple two-dimensional trade-off when the constraint is relaxed (see discussion following Proposition 2). Solution ease is one practical advantage of unrestricted local spending. However, there is a more important consideration. In particular, there is a question on when the elimination of the local spend constraint will, and will not, have a large impact on alleviating human suffering. The question is meaningful because HOs that are tightly constrained on local spend may consider initiatives to mitigate such a constraint (e.g., line of credit underwritten by large donors that is available during the immediate relief period). Such an initiative requires effort on the part of HO management and a compelling case for its value. Our model can be used to make such a case through quantification of value.

Most of these results and insights are derived based on the assumption that reactive stock is less expensive than prepo stock. When the situation is reversed, the main insights remain, but detailed solutions become more complex.

Acknowledgments

The authors thank David Coddington, a Senior Advisor of Global Supply Chain Management Unit at Catholic Relief

Services (CRS), for his exceptional dedication to this study and Rodolphe Peltier, the former Senior Director of Global Supply Chain Management at CRS (currently the Director of Supply Chain at Chemonics International), for his generous support. They also thank Lionel Lajous, an Emergency Operations Technical Advisor at CRS; Sarah Robbins-Penniman, the Director of Supply Chain Performance at CRS; Scott Dubin, a Senior Advisor of Supply Chain for the Office of HIV/AIDS at USAID; and Mary Jelliti, the Logistics Coordinator at GOAL International; for their valuable comments and information that they shared with us throughout this project.

Endnotes

[1] The remaining 4% is the cost of capital.

[2] As a minor technical point, $m_c(x)$ and $m_s(x)$ are proportional to the true marginal prepo cost and savings functions by a probability factor (see the proof of Proposition 5). The same observation applies to the lower- and upper-bound functions. We exclude this proportionality factor in order to facilitate comparison with marginal functions from the static model, and we refer to these functions as marginal cost and savings.

[3] See https://www.emdat.be/ (accessed date March 15, 2019).

[4] See https://glidenumber.net/glide/public/search/search.jsp (accessed date March 15, 2019).

[5] Empirically, a variety of arrival processes show evidence of exponentially distributed interarrival times (e.g., outbreaks of wars, tornadoes (Richardson 1956, Hayes 2002)). Theoretical support for this observation comes from the Khintchine limit theorem: under fairly mild assumptions, the distribution of time between arrivals for a process that is a superposition of n independent arrival processes approaches exponential as n increases (Khintchine 1960, Feller 1965).

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